Supporting Valid-Time Indeterminacy

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In *valid-time indeterminacy* it is known that an event stored in a database did in fact occur, but it is not known exactly *when*. In this paper we extend the SQL data model and query language to support valid-time indeterminacy. We represent the occurrence time of an event with a set of possible instants, delimiting when the event might have occurred, and a probability distribution over that set. We also describe query language constructs to retrieve information in the presence of indeterminacy. These constructs enable users to specify their *credibility* in the underlying data and their *plausibility* in the relationships among that data. A denotational semantics for SQL's select statement with optional credibility and plausibility constructs is given. We show that this semantics is *reliable*, in that it never produces incorrect information, is *maximal*, in that if it were extended to be more informative, the results may not be reliable, and *reduces* to the previous semantics when there is no indeterminacy. Although the extended data model and query language provide needed modeling capabilities, these extensions appear initially to carry a significant execution cost. A contribution of this paper is to demonstrate that our approach is useful and practical. An efficient representation of valid-time indeterminacy and efficient query processing algorithms are provided. The cost of support for indeterminacy is empirically measured, and is shown to be modest. Finally, we show that the approach is general, by applying it to the temporal query language constructs being proposed for SQL3.

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1. OVERVIEW

Most databases record the history of an enterprise. Such databases associate with each event a *timestamp* indicating when that event occurred. Often, a user knows only approximately when an event happened. For instance, she may know that it happened "between 2 PM and 4 PM," "sometime last week," or "around the middle of the month." These are examples of *valid-time indeterminacy*. Information that is valid-time indeterminate can be characterized as "don't know when" information, or more precisely, "don't know *exactly* when" information. This kind of information has various sources, including the following.

- Granularity mismatch—In many cases the granularity with which data is recorded is finer than the precision to which the occurrence time of an event is known. For example, an occurrence time known to within one hour, recorded on a system with timestamps in the granularity of a second, happened sometime *during* that hour, but during which second is unknown.

- *Dating techniques*—Many dating techniques, such as Carbon-14 dating [Goudsmit and Claiborne 1966], are inherently imprecise.

- *Uncertainty in planning*—Projected completion dates are often inexactly specified, e.g., the project will complete three to six months from now.

- *Unknown or imprecise event times*—In general, occurrence times could be unknown or imprecise. For example, perhaps we do not know when a student in the first grade was born. The student's date of birth could be recorded in the database as either unknown (she was born between the beginning and the end of time) or imprecise (she was born between five and seven years ago).

- *Clock measurements*—Every clock measurement has some imprecision [Petley 1991].

This paper adds valid-time indeterminacy to SQL [Melton and Simon 1993]. We identified several design goals to be met in extending SQL with indeterminacy. First, we wanted the syntactic extensions to be minimal, yet highly expressive. Timestamps should include a representation for valid-time indeterminacy and users should be able to control, via query language constructs, the amount of indeterminacy present in derived information. Second, we wanted the extensions to be upwardly compatible with SQL. In the absence of valid-time indeterminacy, the syntax and semantics should reduce to that of SQL. Third, the semantics should be simple and intuitive. In particular, information should not be "invented" during queries. And finally, it was critical that valid-time indeterminacy have little impact on the performance of the DBMS, either in space to store indeterminate timestamps or in query evaluation time.

Unfortunately, these design goals conflict, as discussed further in Section 11. Earlier work required a three-valued [Snodgrass 1982] or four-valued

logic [Schiel 1987]. Probabilistic approaches (e.g., Barbará et al. [1992]) are highly expressive, but have a query evaluation cost that is proportional to the number of probable alternatives. The number of such alternatives can easily number in the millions for a single indeterminate time, and thus probabilistic approaches are impractical if directly applied in a temporal context. More recent work models indeterminacy by using a constraint network. At best, using a restricted network, query evaluation complexity has been shown to be cubic [Brusoni et al. 1995] in the number of constraints. Our approach uses probabilistic weights, yet achieves a linear query evaluation complexity, with a nominal increase in storage cost.

The next section introduces an example requiring the storage of valid-time indeterminate information; this example is used throughout the paper. We then examine the representation of valid-time indeterminacy. After that, we explore what it means to retrieve information from a database with valid-time indeterminacy. We outline syntactic and semantic extensions to SQL to support retrieval of valid-time indeterminate information, and argue their correctness. We then show how valid-time indeterminacy can be implemented. Although retrieving valid-time indeterminate information may appear to be expensive, we demonstrate that an efficient implementation exists. The final sections apply these same notions to a temporal query language, trace related work, summarize our approach, and discuss future work. Proofs for all theorems can be found in the Appendix.

2. MOTIVATING EXAMPLE

An example database is shown in Figure 1. This database models a single company with two warehouses and one airplane factory. The warehouses supply parts to the factory. Each warehouse keeps a *Sent* relation, which records when parts were shipped from the warehouse to the factory. The factory maintains the *In_ Production* relation, which is a production history of airplanes built by the factory. This relation includes a period timestamp[1]; the previous two relations include instant timestamps. For each relation, we assume an underlying timestamp granularity of one day.[2]

Valid-time indeterminacy naturally arises in both base relations and derived relations. The *During* attribute of the *In_ Production* base relation is an indeterminate period. This is because the granularity of the *In_ Production* relation is a month. A month is an indeterminate value that represents a set of possible days. Production on an airplane started on some day in the indicated month, but we cannot be sure which one. For example, production on the Centurion with serial number AB33 started sometime between (inclusive) March 1 and March 31. For this example we assume

[1]This paper uses the SQL/Temporal term *period* [Melton 1996] to denote an anchored duration of time, e.g., the year 1995, in the same way that *interval* is used in SQL to denote an *unanchored* duration of time, e.g., one year, in place of the term *interval* [Jensen et al. 1994], which appears more commonly in the literature.

[2] For expository purposes only, we adopt a non-SQL format for temporal constants, e.g., May 1 rather than 1996-05-01.

Sent_by_Boeing(Lot_Num, Part, When)

Lot_Num	*Part*	*When*
23	wing strut	May 6
24	engine	June 4

Sent_by_Cessna(Lot_Num, Part, When)

Lot_Num	*Part*	*When*
30	wing strut	May 26
31	wing strut	June 9

In_Production(Model, Serial_Num, During)

Model	*Serial_Num*	*During*
Centurion	AB33	[March 1 ~ March 31 – June 1 ~ June 30]
Cutlass	Z19	[June 1 ~ June 30 – July 1 ~ July 31]
Centurion	AB34	[June 1 ~ June 30 – August 1 ~ August 31]
Caravan	FA2K	[April 1 ~ April 30 – May 1 ~ May 31]

Received(Warehouse, Lot_Num, Part, When)

Warehouse	*Lot_Num*	*Part*	*When*	
Boeing	23	wing strut	May 10 ~ May 29	e_1
Cessna	30	wing strut	May 30 ~ June 18	e_2
Boeing	24	engine	June 8 ~ June 27	e_3
Cessna	31	wing strut	June 13 ~ July 2	e_4

Fig. 1. An example database.

that production is equally likely to have started or ended during any day in an indicated month, although in general we allow nonuniform likelihoods.

The *Received* relation is not maintained by either the factory or a warehouse; rather it is a derived relation, the product of educated guesswork. Parts are shipped by truck from a warehouse and arrive at the factory no earlier than 4 and no later than 24 days after they leave a warehouse. The *Received* relation is computed from each warehouse's *Sent* relation by adding a 4–24 day "fudge factor" to the *When* attribute. The valid times in the *Received* relation are indeterminate; that is, we know roughly when the parts were received, but do not know exactly which day they were received. We assume that each day in the recorded range of days is equally likely. For example, the batch of engines received from the Boeing warehouse arrived on one of the days in the set {June 8, June 9, ..., June 27}, but we have no reason to favor one day over another. (The e_i to the right of each instant is for expository purposes only; it is a short label for the instant that is used in other sections of the paper.)

Queries can make use of indeterminate information in the database. Suppose that a few of the Centurion airplane owners report a faulty wing strut. Naturally, we would like to query the database to determine which warehouse(s) supplied the defective parts and, specifically, which lots are implicated (we give such a query in Section 4). In SQL with valid-time

The Definite Answer

Warehouse	*Lot_Num*	*Part*	*When*
Boeing	23	wing strut	May 10 ~ May 29

The Probable Answer

Warehouse	*Lot_Num*	*Part*	*When*
Boeing	23	wing strut	May 10 ~ May 29
Cessna	30	wing strut	May 30 ~ June 18

The Possible Answer

Warehouse	*Lot_Num*	*Part*	*When*
Boeing	23	wing strut	May 10 ~ May 29
Cessna	30	wing strut	May 30 ~ June 18
Boeing	24	engine	June 8 ~ June 27

Fig. 2. Answers to example queries.

indeterminacy, we could query to determine which shipment of wing struts "overlaps" the production of a Centurion airplane. Overlap is the operation of temporal intersection.

There are two well-defined limits on an answer to a query in an incomplete information database: the *definite* answer and the *possible* answer [Lipski 1979]. The definite answer is the information that satisfies the query in *all* possible extensions of the database, while the possible answer is the information that satisfies the query in *some* possible extension of the database (we formalize these bounds in Section 5.3). For example, consider a temporal selection on the *Received* relation in Figure 1 that selects those parts received prior to June 10. Even though the exact date the shipment of lot number 23 from the Boeing warehouse arrived is unknown, it is clear that this shipment arrived before June 10 (the shipment arrived on some day in the set {May 10, May 11, . . . , May 29}). This tuple, and no other, is in the definite answer to the query. Lot number 30 from the Cessna warehouse is in the possible answer to the query. It is possible that this shipment arrived prior to June 10 (and also possible that it did not). Similarly, lot number 24 from the Boeing warehouse possibly arrived prior to June 10. The first shipment from the Boeing warehouse is also in the possible answer because a definite answer is also a possible answer, but not vice-versa.

Between the possible and definite limits lie other answers. For instance, assume that it is equally likely for each day in {June 8, June 9, . . . , June 27} that lot number 24 arrived. For the shipment to have arrived prior to June 10, it had to arrive on either June 8 or June 9. If all the days are considered to be equally likely, then there is a probability of only 0.10 (2 chances out of 20) that the the shipment was received prior to June 10. So it is *improbable* that lot number 24 arrived prior to June 10. However, it is *probable* that both lot number 30 (0.55 probability, 11 chances out of 20) and lot number 23 did arrive (1.00 probability). The definite, possible, and "probable" answers to the temporal selection are shown in Figure 2. (There

is a detailed example of obtaining a "probable" answer for a query in Section 5.7.) If the query language can make use of a probability distribution over the possible times associated with an indeterminate instant, a "richer" query language results, one not restricted to the definite and possible answers. The richness of the query language, however, must not compromise efficient implementation nor detract from the intuitiveness of the language.

There are two stages to determining an answer to a query. The first retrieves the data that is relevant to the query. The second stage constructs an answer that satisfies the conditions specified in the query. We provide separate controls on the indeterminacy for each stage.

Correlation credibility (potentially) changes the information available to query processing by replacing each indeterminate time value with a relevant determinate time value. A typical replacement is the *expected value* or probabilistic mean. For example, the expected start of production for the Centurion with serial number AB33 is March 15 (the expectation of the uniform distribution over a sequence of values is the half-way point in that sequence). In SQL with indeterminacy, the user can express this preference by selecting an appropriate correlation credibility value. The chosen correlation credibility potentially modifies every time value in the associated relation, removing indeterminacy.

Ordering plausibility controls the construction of an answer to the query. For instance, a Centurion owner could query which shipment of wing struts plausibly arrived during production of his or her plane. Intuitively, such a query relaxes the constraints on the relationship between the production times and the day a shipment was received from "do they definitely overlap?" to "is it probable that they overlap?" or perhaps to "is it even remotely possible that they overlap?" The user selects the kind of overlap that she or he requires by setting an appropriate ordering plausibility value. It is probable that lot number 31 from the Cessna warehouse was received during production of the Centurion with serial number AB33, but one cannot be absolutely sure that it did.

There is a natural division between indeterminacy in the data and indeterminacy in the query. The support for valid-time indeterminacy that we add to SQL allows the user to control both. Correlation credibility replaces indeterminacy in the data, while ordering plausibility governs the probability of relationships among the data.

3. EXTENDING THE DATA MODEL WITH INDETERMINACY

In this section we discuss how to represent indeterminate instants, periods, and intervals in the data model. In Section 7 we discuss how these representations are implemented.

3.1 Model of the Time-Line

We briefly summarize the simple, standard model of time that we adopt for this paper. The model is presented in detail elsewhere [Dyreson et al. 1995].

Time has a standard geometric metaphor, in which time itself is a line segment (assuming a bounded universe); a point on the time-line is called an *instant*; the time between two instants is known as a *time period* (period for short); and a length, or unanchored segment, of the time-line is an *interval*. The time-line segment is partitioned into a finite number of smaller segments, each called a *chronon* [Ariav 1986; Clifford and Rao 1987; Jensen et al. 1994]. A chronon is the smallest amount of time that can be represented in the implementation. The chronons are consecutively labeled with the integers in the sequence 0, . . . , N, where N is the number of values that a timestamp can represent.

3.2 Indeterminate Instants

An instant is *determinate* if it is known when (i.e., during which particular chronon) it is located. Often, however, we do not know the exact chronon during which an instant is located; instead, we only know that the instant is located sometime during a set or range of chronons. We call such an instant an *indeterminate* instant.

An indeterminate instant is described by a *lower support*, an *upper support*, and a *probability mass function (p.m.f.)* [Dyreson and Snodgrass 1993]. The supports are chronons that delimit when the instant is located; the instant is no earlier than during the lower support and no later than during the upper support. Between the supports lies a *period of indeterminacy*. The period of indeterminacy is a contiguous set of *possible chronons*. The instant is located during some chronon in this set, but which chronon is unknown. We denote a set of possible chronons that extends from the lower support, α_*, to the upper support, α^*, using the notation $\alpha_* \sim \alpha^*$, e.g., `May 10 ~ May 29`.

3.2.1 *Probability Mass Function.* Although an indeterminate instant is located during some possible chronon, not all the possible chronons are equally likely. For example, it could be that the instant is most likely located during the earliest chronon in the period of indeterminacy. The probability mass function gives the probability of each chronon. The probability mass function, P_α, for the indeterminate instant α is

$$P_\alpha(i) = \mathbf{Pr}[\alpha = i] \; i \in \{0,1, \; \ldots, N\}$$

where $\mathbf{Pr}[\alpha = i]$ is the probability that the instant is located during chronon i. Since the instant is not any time outside the period of indeterminacy, $\mathbf{Pr}[i < \alpha_*] = 0$ and $\mathbf{Pr}[i > \alpha^*] = 0$. All indeterminate instants are considered to be independent, that is,

$$\mathbf{Pr}[\alpha = i \wedge \beta = j] = \mathbf{Pr}[\alpha = i] \times \mathbf{Pr}[\beta = j].$$

Like most other probabilistic approahes in databases [Barbará et al. 1990; 1992; Dey and Sarkar 1996; Cavallo and Pittarelli 1987; Fuhr and Rölleke 1997; Gelenbe and Hebrail 1986; Kornatzky and Shimony 1993a; 1993b;

Zimányi 1992] no provisions are made for joint or dependent probabilities. An indeterminate instant, α, is denoted using the notation $(\alpha_* \sim \alpha^*, P_\alpha)$.

3.2.2 *Mass Function Sources*. The probability mass function for an indeterminate instant is supplied when the instant is created. In many common cases the probability mass function for an indeterminate instant stems from the source of the indeterminacy (the list below is not exhaustive).

- *Granularity mismatch*—The uniform or equiprobable mass function is a useful assumption. For example, an instant known to within one hour and recorded on a system with timestamps in the granularity of a second happened sometime *during* that hour, but during which particular second is unknown, and there is no a priori reason to favor one second over another.
- *Dating techniques*—A property of radioactive dating techniques is that the estimate is described by a normal, "bell-shaped curve" distribution.
- *Uncertainty in planning* —Analysis of past data (the past data may be readily available in a temporal database) can sometimes provide a good indicator of future performance. For instance, we may not know exactly when an airline will depart. However, an analysis of past departure times for that route, type of airline, and day of the week (the analysis could be much more elaborate) may show that this flight tends to leave later than scheduled. Based on this analysis, a "probably late" distribution could be used for the departure time of that flight.
- *Unknown or imprecise instants*—Typically, if the location of an instant is unknown, the distribution is also unknown. In these situations a user can specify that the distribution is *missing*; see below.
- *Clock measurements*—Clock-specific distributions model the imprecision of specific clock measurements [Petley 1991].

Dey and Sarkar [1996] provide several additional means of determining the underlying mass function.

In some cases a user just may not know the underlying mass function because that information is unavailable or the mass function might exceed the implementation capacities of the system (Section 8 describes the implementation and the constraints it imposes on mass functions). In such cases the distribution can be specified as missing. A distribution that is missing represents a complete lack of knowledge about the distribution. It is a kind of second-order incompleteness, that is, the distribution that is missing is incomplete information about indeterminate information. Unlike some other probabilistic data models [Barbará et al. 1990; 1992], we do not allow partially known distributions.

While the terminology introduced so far suggests a difference between indeterminate and determinate instants, it is instructive to note that an indeterminate instant can be used to model a determinate instant. A

determinate instant can be modeled by an indeterminate instant with a singleton set of possible chronons. A determinate instant records that an instant is located sometime *during* a particular chronon. Without loss of generality, we assume that a determinate instant represents any real-world instant during a chronon. Hence, at an abstract level, the *exact* real-world instant modeled by a determinate instant is never precisely known. At best, only the chronon during which it is located is known.

3.3 Indeterminate Periods and Intervals

A *determinate period* is the time between two determinate instants.[3]

A period bounded by indeterminate instants (called the *starting* and *terminating instants*) is termed an *indeterminate period* [Dyreson 1994]. An indeterminate period could start during any member of the set of possible chronons of the starting instant. Likewise, the indeterminate period could end during any member of the set of possible chronons of the terminating instant. Since the location of the starting and terminating instants are known only imprecisely, it follows that it is unknown precisely when an indeterminate period begins or ends. We assume that the starting instant must come before the terminating instant in the period, that is, the bounding instants can overlap on at most a single chronon.[4]

A *determinate interval* is a precisely known duration of time, e.g., six days, and is represented as a count of chronons. An *indeterminate interval*, on the other hand, is an imprecise duration that describes a set of possible durations. An indeterminate interval is represented by an imprecise number of chronons, e.g., "from two to three chronons." The representation of an indeterminate interval has an associated probability mass function that gives the likelihood of each possible duration.

We now turn from the data model to the query semantics.

4. SYNTACTIC EXTENSIONS TO SQL

In this section, we summarize the syntactic extensions to SQL to support the storage and retrieval of valid-time indeterminate information from a database. Full coverage of the syntactic extensions can be found elsewhere (with slight modifications) [Dyreson and Snodgrass 1995b; Snodgrass 1995]. In the next section, we provide a formal semantics for these constructs. We separate the syntax from the semantics to emphasize that the syntactic extensions are minor.

Four syntactic extensions to SQL are needed to support valid-time indeterminacy: (1) to indicate that a temporal attribute is indeterminate,

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[3]While the period data type is not in SQL-92 [Melton and Simon 1993], it is included in the SQL/Temporal part of SQL3 [Melton 1996]. In our model of time, it is represented by a sequence of chronons, denoted by the starting and terminating chronons in the sequence.

[4]In a temporal query language (see Section 10), this assumption can be relaxed. Periods are dynamically constructed from a pair of underlying instants, which may overlap, by a *period constructor* during a query. The period constructor can check to ensure that the starting instant is before the terminating instant to the specified plausibility and credibility values.

(2) to specify the correlation credibility, (3) to specify the ordering plausibility, and (4) to indicate that a temporal literal is indeterminate.

The first syntactic extension, to indicate that an attribute is indeterminate, involves schema specification statements. In the create table statement, a user may add either the modifier INDETERMINATE or INDETERMINATE COMPACT before an instant, period, or interval attribute specification to specify that the value may be indeterminate. The two modifiers toggle between alternative storage strategies for indeterminate timestamps, discussed in Section 7 (the compact version is a less expressive, smaller timestamp). We also add an optional "with" phrase to the end of an attribute's specification to allow the user to specify *standard* or *nonstandard* mass functions. These two categories of mass functions are also discussed in Section 7 (the standard functions have a more compact representation). The default is WITH STANDARD DISTRIBUTION. For periods, modifiers apply to both bounding instants in the period. Consistent with SQL's handling of NULL values, an indeterminate temporal attribute may not be included in a key. We provide clauses to the alter table statement that allow any of these aspects to be changed. Below are some examples of the extended create table and alter table statements that describe the relations given in Figure 1.

```
CREATE TABLE Received(Warehouse  CHARACTER(30),
                      Lot_Num    INTEGER,
                      Part       CHARACTER(40),
                      When       INDETERMINATE DATE);
CREATE TABLE In Production(Model CHARACTER(30),
                      Serial_Num CHARACTER(10),
                      During     INDETERMINATE PERIOD(DATE));
ALTER TABLE Received ALTER COLUMN When
   TO NONSTANDARD DISTRIBUTION;
```

The second syntactic extension supports correlation credibility. The from clause in the select statement declares the relations over which the query is to be evaluated and associates correlation name(s) to each relation. The correlation credibility is denoted via a WITH CREDIBILITY phrase. Credibility is a *replacement strategy* for each indeterminate instant, interval, and period–bounding instant in the correlated relation. The credibility phrase sets the credibility to one of four possible strategies below.

1. INDETERMINATE—Retain all indeterminacy; do not replace any time values.

2. EXPECTED—Replace each indeterminate time value with the expected value, i.e., the probabilistic mean. This will compute the expected result for a query.

3. MAX—Replace each indeterminate time value with the lower support (except for an instant that starts a period, in which case use the upper support). For periods, this value eliminates all the indeterminacy. For instants, it uses the earliest possible instant; while for intervals, the

shortest possible interval is chosen. Intuitively, this option "maximizes determinacy" ; credibility is at a maximum when the indeterminacy is at a minimum.

4. MIN—Replace each indeterminate time value with the upper support (except for an instant that starts a period, in which case use the lower support). For periods, this value converts all the indeterminate information to determine information. For instants, it chooses the latest possible instant; while for intervals, the longest possible interval is used. Intuitively, this option "minimizes determinacy" ; credibility is at a minimum when indeterminacy is at a maximum.

The credibility phrase is optional and has an initial default value of INDETERMINATE. The default value can be changed using a SET DEFAULT CREDIBILITY statement. This support for credibility simplifies that described elsewhere [Dyreson and Snodgrass 1993; Dyreson 1994; Dyreson and Snodgrass 1995b].

The third syntactic extension concerns ordering plausibility. The ordering plausibility is the plausibility in the where predicate among the instants, periods, and intervals that participate in the predicate. The default ordering plausibility is specified using a SET DEFAULT PLAUSIBILITY statement. The default ordering plausibility can be overridden in a select statement by appending a WITH PLAUSIBILITY phrase to the end of the where clause. The ordering plausibility is an integer value between 1 and 100 (inclusive), and has an initial default value of 100. An ordering plausibility of 1 indicates that any possible answer is desired (i.e., the where predicate can be satisfied by any possible extension); an ordering plausibility of 100 requests the definite answer (i.e., the where clause must be satisfied by all possible extensions).

The final syntactic extension is to support indeterminate temporal literals. All indeterminate literals follow the convention that that we have used heretofore in this paper, namely, indeterminate time is represented as a range of times separated by a '~', e.g., DATE '5/10/1997 ~ 5/29/1997' represents the indeterminate instant May 10, 1997 ~ May 29, 1997. A probability mass function can be *named* in the literal, e.g., DATE '5/10/1997 ~ 5/29/1997 UNIFORM' represents an instant with a uniform probability mass function; the default probability mass function is *missing*.

An example query illustrating the various constructs is shown in Figure 3. Intuitively, the query determines, within the specified plausibility and credibility levels, which wing strut shipments were received during production of each Centurion. The from clause specifies that all information, regardless of its credibility, from the *In_Production* relation should be used (via the specified credibility of INDETERMINATE). The where clause selects pairs of Centurion and wing strut tuples that overlap with a plausibility of 60. Finally, the target list determines when the shipment of possibly defective parts was received. When this query is applied to the database

```
SET DEFAULT PLAUSIBILITY 60
SELECT r.Warehouse, r.Lot_Num, p.Serial_Num, r.When
   FROM Received AS r WITH CREDIBILITY INDETERMINATE,
        In_Production AS p WITH CREDIBILITY INDETERMINATE
   WHERE p.Model = "Centurion" AND r.Part = "wing strut"
         AND r.When OVERLAPS p.During
```

Fig. 3. An example query.

Warehouse	*Lot_Num*	*Serial_Num*	*When*		
Boeing	23	AB33	May 10	~	May 29
Cessna	30	AB33	May 30	~	June 18
Cessna	31	AB34	June 13	~	July 2

Fig. 4. Result of the example query.

shown in Figure 1, the relation shown in Figure 4 is computed. In Section 5.7, we discuss in detail how this query is evaluated to obtain the indicated result.

5. SEMANTIC EXTENSIONS TO SQL

In this section, we extend the semantics of SQL to support indeterminacy. The presentation focuses on SQL's select statement. We first provide a brief review of the semantics of the select statement. We then extend the semantics to support indeterminacy. The evaluation of a select in the extended semantics has a possible and a definite interpretation, as well as other interpretations that lie between those bounds. We show that the possible interpretation is both *reliable* , in that it does not invent information, and *maximal*, insofar as it cannot be strengthened to produce more results. However, the indeterminate semantics does not demonstrate that indeterminacy can be efficiently implemented. Consequently, we introduce an operational semantics. The operational semantics provides all the necessary support for indeterminacy with three changes to the SQL semantics. First, it redefines the temporal ordering relation, *Before*. Second, it introduces a 4-sorted domain for the evaluation of where clause predicates. And third, it adds a *Replace* operator to effect correlation credibility. Each of the changes incorporates the determinate semantics (as the default). Hence, the semantics of existing queries is left unchanged. We show that operational semantics correctly implements indeterminate semantics.

5.1 Review of SQL Semantics

In this section, we present a simplified semantics for the select statement. Our goal is to highlight those aspects of the statement that will be impacted by valid-time indeterminacy, a theme that we develop in subsequent sections.

We use the notation, $[\![x]\!]_{SQL}$, to denote the meaning of the syntactic SQL construct x. The top-level of denotational semantics for the select statement, applied to a database d, is given below.

$$[\![\texttt{SELECT}\ \langle \mathit{target\ list} \rangle\ \texttt{FROM}\ \langle \mathit{from\ list} \rangle\ \texttt{WHERE}\ \langle \mathit{predicate} \rangle]\!]_{SQL}(d)$$

$$= [\![\langle \mathit{target\ list} \rangle]\!]_{SQL}([\![\texttt{WHERE}\ \langle \mathit{predicate} \rangle]\!]_{SQL}([\![\langle \mathit{from\ list} \rangle]\!]_{SQL}(d)))$$

The select statement first applies the from clause to the database. This clause computes the Cartesian product of the relations specified in the $\langle \mathit{from\ list} \rangle$. The meaning of the from clause is

$$[\![\langle \mathit{from\ list} \rangle]\!]_{SQL}(d) = [\![\langle \mathit{relation}_1 \rangle,\ \ldots,\ \langle \mathit{relation}_n \rangle]\!]_{SQL}(d)$$

$$= [\![\langle \mathit{relation}_1 \rangle]\!]_{SQL}(d) \times\ \ldots\ \times [\![\langle \mathit{relation}_n \rangle]\!]_{SQL}(d)$$

where

$$[\![\langle \mathit{relation} \rangle]\!]_{SQL}(d) = r_i,\ r_i \in d,\ \text{and } r_i \text{ is named } \langle \mathit{relation} \rangle.$$

The result computed by the from clause, an intermediate relation r, is then used as an argument for the where clause. This clause selects those tuples that satisfy the *predicate* in the where clause.

$$[\![\texttt{WHERE}\ \langle \mathit{predicate} \rangle]\!]_{SQL}(r) = \{t \mid t \in r \wedge [\![\langle \mathit{predicate} \rangle]\!]_{SQL}(t)\}.$$

Note that the correlation names appearing in $\langle \mathit{predicate} \rangle$ need to be mapped to the attributes in the tuple t. This is generally done by passing the symbol table as a second argument to $[\![\,]\!]_{SQL}$. For simplicity, we omit that argument. We restrict the presentation to predicates that are logical formulas constructed from comparison operations and Boolean connectives, and rely on the readers' background knowledge of SQL to supply the meaning of each SQL $\langle \mathit{predicate} \rangle$, i.e., the standard Boolean logic applies:

$$[\![\langle \mathit{predicate} \rangle\ \texttt{AND}\ \langle \mathit{predicate} \rangle]\!]_{SQL}(t) =$$

$$[\![\langle \mathit{predicate} \rangle]\!]_{SQL}(t) \wedge [\![\langle \mathit{predicate} \rangle]\!]_{SQL}(t).$$

In the final step, the output of the where is projected onto the desired domains specified by the $\langle \mathit{target\ list} \rangle$.

$$[\![\langle \mathit{target\ list} \rangle]\!]_{SQL}(r) = \pi_{[\![\mathit{target\ list}]\!]_{SQL}}(r).$$

Here we also ignore the mapping of correlation names to attributes of r.

5.2 Supporting Syntactic Extensions

Since the current SQL semantics does not support indeterminacy, the extended syntax can only be supported in a new, extended semantics,

which we denote $[\![]\!]_{ind}$. The extensions presented in Section 4 include two additional controls on indeterminate information: correlation credibility and ordering plausibility. These values appear as additional parameters, δ and γ, respectively, to $[\![]\!]_{ind}$. Both values can be specified using SET DEFAULT statements.The meanings of the SET DEFAULT statements are given below. Assume S is any SQL statement.

$$[\![\texttt{SET DEFAULT CREDIBILITY } \delta'; S]\!]_{ind}(\delta, \gamma, d) = [\![S]\!]_{ind}(\delta, \gamma, d)$$

$$[\![\texttt{SET DEFAULT PLAUSIBILITY } \gamma'; S]\!]_{ind}(\delta, \gamma, d) = [\![S]\!]_{ind}(\delta, \gamma', d)$$

The default values can be overridden within the select statement itself. We show here how to override the plausibility default and in Section 5.6 how to override the credibility default.

$$\begin{aligned}&[\![\texttt{SELECT } \langle \textit{target list} \rangle \texttt{ FROM } \langle \textit{from list} \rangle\\ &\qquad \texttt{WHERE } \langle \textit{predicate} \rangle \texttt{ WITH PLAUSIBILITY } \gamma']\!]_{ind}(\delta', \gamma, d) =\\ &[\![\texttt{SELECT } \langle \textit{target list} \rangle \texttt{ FROM } \langle \textit{from list} \rangle \texttt{ WHERE } \langle \textit{predicate} \rangle]\!]_{ind}(\delta, \gamma', d)\end{aligned}$$

The initial default credibility is INDETERMINATE and initial default plausibility is 100.

5.3 An Overview of Indeterminate Semantics

In this section, we give an overview of indeterminate semantics and discuss several properties that a semantics involving incomplete information should possess. In later sections, we show that our semantics does indeed have these essential properties.

The indeterminate semantics for the select statement is outlined below. This semantics has the same structure as SQL semantics.

$$\begin{aligned}&[\![\texttt{SELECT } \langle \textit{target list} \rangle \texttt{ FROM } \langle \textit{from list} \rangle \texttt{ WHERE } \langle \textit{predicate} \rangle]\!]_{ind}(\delta, \gamma, d)\\ &\qquad = [\![\langle \textit{target list} \rangle]\!]_{ind}(\gamma, [\![\texttt{WHERE } \langle \textit{predicate} \rangle]\!]_{ind}(\gamma, [\![\langle \textit{from list} \rangle]\!]_{ind}(\delta, d)))\end{aligned}$$

SQL and indeterminate semantics differ in two ways. First, the indeterminate semantics has additional parameter(s), but note that δ is utilized in the where predicate and target list only. Second, the select statement has a different meaning in the indeterminate semantics. Let us consider what the meaning should be, intuitively.

A select statement applied to a database containing only complete information and evaluated under SQL semantics has a single interpretation. In contrast, a retrieval from a database containing incomplete information has at least two interpretations. One interpretation is that the query selects information that *possibly* matches the retrieval constraints. The second interpretation is that the query selects information that *definitely* matches the retrieval constraints. Which interpretation is adopted is specified by the user via syntactic constructs in the query. It is

important, however, to guarantee that a query will not produce *impossible* results. That is, a query should be constrained to compute a subset of the possible interpretation and a superset of the definite interpretation. We formalize this by introducing the concept of a *completion*.

An indeterminate instant can be thought of as a set of possible instants, one of which is the "actual" instant, but which one is unknown. Each of the possible instants represents a different, complete description of reality. Each possibility is termed a completion of the instant.[5] The following definition captures this intuition.

Completion of an indeterminate instant. Let $\alpha = (\alpha_* \sim \alpha^*, P_\alpha)$. A completion of an indeterminate instant α is α_i, where $\alpha_i$is a determinate instant such that $\alpha_* \leq \alpha_i \leq \alpha^*$. The set of all completions for an instant α is denoted $\mathbf{C}(\alpha)$.

Completions of periods and intervals also exist. A completion of an indeterminate period is one in which its delimiting indeterminate instants are both replaced by their completions. A completion of an indeterminate interval is one of the possible durations. The concept can also be generalized to apply to tuples, relations, and databases. A completion of x, be it a tuple, relation, or database, is x_c, where x_c is the same as x but, with each indeterminate instant, period, and interval, replaced by a completion of that value. The set of all completions for an entity x is denoted $\mathbf{C}(x)$.

In indeterminate semantics, the possible interpretation of the select statement is attained by using a plausibility of 1, while the definite interpretation is given by adopting a plausibility of 100. We focus on the where clause to show the differences in these two interpretations. The definite interpretation of the where clause is given below.

$$[\![\texttt{WHERE } \langle predicate\rangle]\!]_{ind}(100, r)$$
$$= \{t \mid t \in r \wedge \forall t' \in \mathbf{C}(t)([\![\langle predicate\rangle]\!]_{SQL}(t'))\}$$

The definite interpretation selects only those tuples that are selected by the SQL semantics (note the use of $[\![\langle predicate\rangle]\!]_{SQL}$) in *every* completion of the tuple.

The possible interpretation differs only slightly. It selects only those tuples that are selected by the SQL semantics in *some* completion of the tuple.

$$[\![\texttt{WHERE } \langle predicate\rangle]\!]_{ind}(1, r)$$
$$= \{t \mid t \in r \wedge \exists t' \in \mathbf{C}(t)([\![\langle predicate\rangle]\!]_{SQL}(t'))\}$$

[5] Gadia et al. [1992] introduced the completion of a temporal tuple and of a temporal relation. Our definition extends this notion to indeterminate instants, periods, and intervals and to conventional tuples, relations, and databases containing such values.

This semantics is *reliable* in the sense that it never produces incorrect information.[6] For a semantics to be reliable, the result of a where clause on a completion of a relation r in the SQL semantics should be consistent with the result in the indeterminate semantics on the relation r.

THEOREM 1. $[\![\text{WHERE } \langle predicate \rangle]\!]_{ind}(1, r)$ *is reliable, that is, for any where clause W,*

$$\forall r' \in \mathbf{C}(r)[[\![W]\!]_{SQL}(r') \in \mathbf{C}([\![W]\!]_{ind}(1, r))].$$

Proofs are given in the Appendix.

The semantics is also *maximal*, in that if the semantics were extended to be more informative, that is, allow more completions, then the result may no longer be reliable. From the previous theorem, we know that $[\![W]\!]_{ind}(1, r)$ contains all the needed completions. We need to determine that it contains no extraneous completions.

THEOREM 2. $[\![\text{WHERE } \langle predicate \rangle]\!]_{ind}(1, r)$ *is maximal, that is, for any where clause, W,*

$$\forall c \in \mathbf{C}([\![W]\!]_{ind}(1, r))[\exists r' \in \mathbf{C}(r)(c = [\![W]\!]_{SQL}(r'))].$$

Note that these two theorems in concert demonstrate that, for all where clauses W and indeterminate relations r,

$$\mathbf{C}([\![W]\!]_{ind}(1, r)) = \bigcup_{r' \in \mathbf{C}(r)} [\![W]\!]_{SQL}(r').$$

Observe that if the database has only complete information, there is only one completion, and effectively just a single interpretation of a query, since the possible and definite interpretations are equivalent. So credibility and plausibility have no effect whatsoever on existing databases, which contain only complete information, and the semantics of extant SQL queries and databases is unchanged by extensions to support indeterminacy. This important property is termed *temporal upward compatibility* [Bair et al. 1997]. In the context of indeterminacy, it is also equivalent to stating that the indeterminate data model is a *generalization* [Gadia et al. 1992] of the determinate (SQL) data model.

The possible and definite interpretations are just two of the many interpretations available in the indeterminate semantics. Other interpretations result from choosing other credibility and plausibility values. These interpretations are related. Increasing the plausibility setting in a select statement yields a "more definite" interpretation. It is essential, however, to ensure that these other interpretations do not generate spurious results. We guarantee this by demonstrating (in Section 5.8) that evaluating a select statement in the indeterminate semantics is monotonic in plausibil-

[6]Here we adopt the notions of reliability, and later, maximality, used by Gadia et al. [1992].

ity. Each result at a higher plausibility setting is a subset of the result at a lower plausibility.

5.4 Supporting Ordering Plausibility

Both the possible and definite interpretations of a query in the indeterminate semantics are stated in terms of SQL semantics. Unfortunately, a straightforward implementation of these interpretations would be highly impractical, since it would require computing the predicate over every possible completion of a tuple in the inner loop of query processing. There could be many completions for each tuple, depending on the duration of the period of indeterminacy. As an example, there are over seven billion completions of a single indeterminate period that has delimiting instants with a period of indeterminacy of one day, assuming a chronon size of one second.

Consequently, we introduce an operational semantics, $[\![]\!]_{op}$, that implements the indeterminate semantics. We show later that the operational semantics is efficient, satisfies the goals outlined above, and is fully consistent with indeterminate semantics, $[\![]\!]_{ind}$.

5.4.1 *Probabilistic Ordering*. The ordering plausibility (primarily) impacts the meaning of the where clause. The semantics of the where predicate without indeterminacy is based on a well-defined ordering of the instants in the underlying relations [Allen 1983]. Every temporal predicate (e.g., OVERLAPS) refers to the ordering given by *Before* to determine the truth value of the predicate. For example, consider the following OVERLAPS predicate.

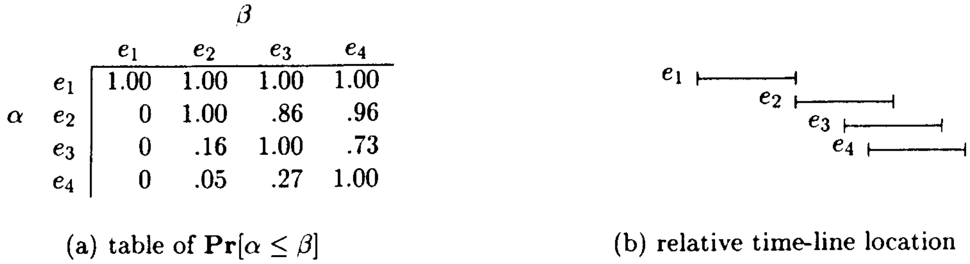
$$[\![\langle instant_1\rangle \texttt{ OVERLAPS PERIOD}(\langle instant_2\rangle, \langle instant_3\rangle)]\!]_{SQL} =$$

$$Before([\![\langle instant_2\rangle]\!]_{SQL}, [\![\langle instant_1\rangle]\!]_{SQL}) \wedge$$

$$Before([\![\langle instant_1\rangle]\!]_{SQL}, [\![\langle instant_3\rangle]\!]_{SQL})$$

The truth value of the predicate depends on the outcome of the *Before* operation.

A set of *determinate* instants has a single temporal ordering. Given a temporal expression consisting of temporal predicates, this ordering either satisfies the expression or fails to satisfy it. A set of indeterminate instants, however, typically has many possible temporal orderings, due to the many completions of those instants. Some of these temporal orderings are plausible, while others are implausible. The user specifies which orderings are plausible by setting an appropriate ordering plausibility value. We stipulate that a temporal expression is satisfied if there exists a plausible ordering between pairs of instants that satisfies each predicate in the expression. This semantics reduces to that of the determinate case when there is only one ordering.



		β			
		e_1	e_2	e_3	e_4
α	e_1	1.00	1.00	1.00	1.00
	e_2	0	1.00	.86	.96
	e_3	0	.16	1.00	.73
	e_4	0	.05	.27	1.00

(a) table of $\mathbf{Pr}[\alpha \leq \beta]$

(b) relative time-line location

Fig. 5. $\mathbf{Pr}[\alpha \leq \beta]$ for the indeterminate instants in *Received*.

In the SQL semantics, *Before* is the "≤" relation on the representation of instants (i.e., on chronons).

$$Before(\alpha, \beta) = \alpha \leq \beta.$$

In operational indeterminate semantics, temporal ordering is, instead, based on the probability that one instant is before another. We define that probability first and then give the new ordering operation.

Probabilistic ordering. For any two indeterminate instants, α and β, the probability that α is before β is

$$\mathbf{Pr}[\alpha \leq \beta] = \sum_{\substack{i,j \in \{0, \ldots, N\} \\ \wedge\, i \leq j}} \mathbf{Pr}[\alpha = i] \times \mathbf{Pr}[\beta = j].$$

Figure 5(a) shows the value of $\mathbf{Pr}[\alpha \leq \beta]$ (to two decimal places) for each pair of instants in the relation *Received*, for example, $\mathbf{Pr}[e_2 \leq e_3] = .86$ (each instant has a uniform p.m.f.). Those instants are placed on a timeline in Figure 5(b).

Probabilistic ordering assumes that there are no dependencies between the probabilities associated with indeterminate instants. Hence, it cannot be used to compute the probability of orderings such as $\mathbf{Pr}[(\alpha \leq \beta \leq \eta)]$, accurately.

Probabilistic equals can be defined analogously.

Probabilistic equals. For any two indeterminate instants, α and β, the probability that α is equal to β is

$$\mathbf{Pr}[\alpha = \beta] = \sum_{i \in \{0, \ldots, N\}} \mathbf{Pr}[\alpha = i] \times \mathbf{Pr}[\beta = i].$$

To handle indeterminate instants in a flexible manner, we define a new operator, $Before_I$, that includes an additional parameter, the ordering plausibility, γ. The value of γ can be any integer between 1 and 100 (inclusive). In general, higher (closer to 100) ordering plausibilities stipulate that only highly probable orderings be considered plausible.

Indeterminate before. For a pair of indeterminate instants, α and β, and a plausibility γ,

$$\mathit{Before}_I(\alpha, \beta, \gamma) = \{\mathit{True} \mid \mathbf{Pr}[\alpha \leq \beta] \times 100 \geq \gamma\} \cup \{\mathit{False} \mid \mathbf{Pr}[\beta < \alpha] \times 100 \geq \gamma\}.$$

In operational semantics, Before_I is defined on the power set of the standard Boolean domain (a four-sorted domain, cf., [Belnap 1977]). There are four possible outcomes for Before_I on a pair of instants. In the first case, the relationship does not hold, nor does its negation, and Before_I evaluates to the empty set. In the second, the relationship holds but its negation does not and Before_I evaluates to {*True*}. In the third case, the relationship does not hold but its negation does. Before_I evaluates to {*False*}. The fourth case covers the other possibility, that both the relationship and its negation hold simultaneously. Before_I evaluates to {*True, False*}.

Note that in operational semantics Before_I treats ordering probabilities between 0 and 0.01 as 0. That is, it treats two instants that have a small chance of occurring before each other as well-ordered in time. To distinguish the well-ordered case from this other case, we define the ordering probability to be 0.01 whenever its value is between 0 and 0.01.

Refined definition of probabilistic ordering. Let α and β be a pair of indeterminate instants.

$$\mathbf{Pr}'[\alpha \leq \beta] = \begin{cases} \mathbf{Pr}[\alpha \leq \beta] & \text{if } \mathbf{Pr}[\alpha \leq \beta] > 0.01 \\ 0 & \text{if } \mathbf{Pr}[\alpha \leq \beta] = 0 \\ 0.01 & \text{otherwise} \end{cases}$$

This function replaces **Pr** in the definition of Before_I. With this substitution, an ordering plausibility of 1 suffices to evaluate every possible ordering, however improbable.

The *missing* mass function is treated as a special case. If one mass function is treated specially and if one (or more) of the instants being ordered has the missing mass function, then the mass is assumed to be distributed in such a way that there is a small, but non-zero, probability ϵ for ordering the two instants. For example, if we introduce an instant e_5 that has an overlapping period of indeterminacy with e_2, but a missing mass function, then $\mathbf{Pr}'[e_2 \leq e_5] = \epsilon$ and $\mathbf{Pr}'[e_5 \leq e_2] = \epsilon$. Consequently, in semantics, an instant with a missing mass function behaves exactly like a *null value*, in that we stipulate that the participation of such an instant in a Before_I operation makes the Before_I evaluate to the empty set (for all plausibilities greater than 1). However, since there is a small probability that an instant with a missing mass is before another instant (when their periods of indeterminacy overlap), Before_I will return {*True,False*} for an ordering plausibility of 1.

Other ordering operations can be defined similarly to Before_I. For instance, we can define Equals_I using probabilistic equals, that is, $\mathbf{Pr}'[\alpha = \beta]$ and $\mathit{Strictly_Before}_I$ using probabilistic less than, that is, $\mathbf{Pr}'[\alpha < \beta]$, etc.

5.4.2 *Predicates and Logical Formulas*. We are now in a position to supply operational semantics for the where clause predicate. We assume, without loss of generality, that this predicate is a logical formula composed of *Before* operations on pairs of instants; a complete set of temporal predicates can be constructed with such formulas [Allen 1983].

$$[\![\langle pred_1\rangle \text{ AND } \langle pred_2\rangle]\!]_{op}(\gamma, r) = [\![\langle pred_1\rangle]\!]_{op}(\gamma, r) \cap [\![\langle pred_2\rangle]\!]_{op}(\gamma, r)$$

$$[\![\langle pred_1\rangle \text{ OR } \langle pred_2\rangle]\!]_{op}(\gamma, r) = [\![\langle pred_1\rangle]\!]_{op}(\gamma, r) \cup [\![\langle pred_2\rangle]\!]_{op}(\gamma, r)$$

$$[\![\text{NOT } \langle predicate\rangle]\!]_{op}(\gamma, r) = \{x \mid \neg x \in [\![\langle predicate\rangle]\!]_{op}(\gamma, r)\}$$

Below, we illustrate the semantics with several examples, using the instants in Figure 5.

$$[\![e_2 \le e_3]\!]_{op}(100, r) = Before_I(e_2, e_3, 100) = \{\}$$
$$[\![e_2 \le e_3]\!]_{op}(50, r) = \{True\}$$
$$[\![e_2 \le e_3]\!]_{op}(1, r) = \{True, False\}$$
$$[\![e_2 \le e_3 \text{ AND } e_1 \le e_4]\!]_{op}(100, r) = \{\}$$
$$[\![e_2 \le e_3 \text{ AND } e_1 \le e_4]\!]_{op}(50, r) = \{True\}$$
$$[\![e_2 \le e_3 \text{ AND } e_1 \le e_4]\!]_{op}(1, r) = \{True\}$$
$$[\![e_2 \le e_3 \text{ OR } e_1 \le e_4]\!]_{op}(100, r) = \{True\}$$
$$[\![e_2 \le e_3 \text{ OR } e_1 \le e_4]\!]_{op}(50, r) = \{True\}$$
$$[\![e_2 \le e_3 \text{ OR } e_1 \le e_4]\!]_{op}(1, r) = \{True, False\}$$
$$[\![\text{NOT } (e_2 \le e_3)]\!]_{op}(100, r) = \{\}$$
$$[\![\text{NOT } (e_2 \le e_3)]\!]_{op}(50, r) = \{False\}$$
$$[\![\text{NOT } (e_2 \le e_3)]\!]_{op}(1, r) = \{True, False\}$$
$$[\![\text{NOT } (e_2 \le e_3 \text{ AND } e_1 \le e_4)]\!]_{op}(1, r) = \{\}$$
$$[\![\text{NOT } (e_2 \le e_3 \text{ AND } e_1 \le e_4)]\!]_{op}(50, r) = \{False\}$$
$$[\![\text{NOT } (e_2 \le e_3 \text{ AND } e_1 \le e_4)]\!]_{op}(1, r) = \{False\}$$
$$[\![\text{NOT } (e_2 \le e_3 \text{ OR } e_1 \le e_4)]\!]_{op}(100, r) = \{False\}$$
$$[\![\text{NOT } (e_2 \le e_3 \text{ OR } e_1 \le e_4)]\!]_{op}(50, r) = \{False\}$$
$$[\![\text{NOT } (e_2 \le e_3 \text{ OR } e_1 \le e_4)]\!]_{op}(1, r) = \{True, False\}$$

A key difference between operational and SQL semantics is that operational semantics uses $Before_I$ rather than *Before*. Here is the operational semantics for the OVERLAPS example given previously.

$$[\![\langle instant_1\rangle \text{ OVERLAPS PERIOD } (\langle instant_2\rangle, \langle instant_3\rangle)]\!]_{op}$$

$$= Before_I([\![\langle instant_2\rangle]\!]_{SQL}, [\![\langle instant_1\rangle]\!]_{SQL}, \gamma)$$

$$\cap\ Before_I([\![\langle instant_1\rangle]\!]_{SQL}, [\![\langle instant_3\rangle]\!]_{SQL}, \gamma)$$

We reiterate that in the probabilistic ordering all instants are assumed to be independent. So in the expression "$Before_I(e_2, e_3, \gamma)$ AND $Before_I(e_3, e_4, \gamma)$" the two $Before_I$ operations are evaluated separately, returning a set of Boolean values that is subsequently intersected. While the strategy of separate evaluation of conjuncts is consistent with determinate semantics, it is important to realize that it is not equivalent to computing $(\mathbf{Pr}[(e_2 \le e_3 \le e_4)] \times 100) \ge \gamma$.

5.4.3 *Indeterminate Semantics of the Where Clause*. The meaning of the where clause in indeterminate semantics can now be given in terms of operational semantics.

$$[\![\text{WHERE } \langle predicate\rangle]\!]_{ind}(\gamma, r) = \{t \mid t \in r \wedge True \in [\![\langle predicate\rangle]\!]_{op}(\gamma, t)\}.$$

Table I. Replacement Strategies

	instant	period		interval
		start	end	
INDETERMINATE	α	α	α	α
EXPECTED	$\mathbf{E}[\alpha]$	$\mathbf{E}[\alpha]$	$\mathbf{E}[\alpha]$	$\mathbf{E}[\alpha]$
MIN	α_*	α_*	α_*	α_*
MAX	α^*	α^*	α^*	α^*

Note that computation of the predicate over the completion of each underlying tuple has been replaced with set operations over the result of the $\mathbf{Pr}'$ function. Admittedly, this function also appears to be expensive to compute, as it is $O(m^2)$ per tuple, where m is the number of chronons in the periods of indeterminacy of the instants to which $\mathbf{Pr}'$ is applied (m can be very large). However, we show in Section 8 that the $\mathbf{Pr}'$ computation can be approximated in constant time. With this result, operational semantics for the evaluation of a temporal expression consisting of temporal predicates and Boolean connectives has the same complexity as that of determinate semantics: $O(n)$, where n is the number of predicates in the query, independent of both the number of chronons and the number of completions of indeterminate values of the tuple.

5.5 The Target List

The indeterminate semantics of the target list is quite simple.

$$[\![\langle target\ list\rangle]\!]_{ind}(\gamma, r) = \pi_{[\![\langle target\ list\rangle]\!]_{ind}(\gamma)}(r)$$

5.6 Supporting Correlation Credibility

Correlation credibility extends the from clause with an optional credibility phrase. In general, correlation credibility is used to replace indeterminate time values with determinate time values. The replacement strategy to use depends upon the credibility value and whether the time value replaced is an instant, period, or interval. Below we define a *Replace* function that effects the replacement for every time value in a tuple.

Replace. Let tuple $t = (\overline{X}, \alpha_1, \ldots, \alpha_n)$ where $\overline{X}$ are non-temporal values and $\alpha_1, \ldots, \alpha_n$ are time values. Then

$$Replace(\delta, t) = (\overline{X}, \mathbf{R}(\alpha_1), \ldots, \mathbf{R}(\alpha_n)).$$

$\mathbf{R}$ is the replacement strategy. Table I lists the replacement strategies for each combination of credibility value and kind of time value, assuming that the time value is represented as $(\alpha_* \sim \alpha^*, P_\alpha)$—note that this representation suffices for instants, period-bounding instants, and intervals—and that $\mathbf{E}[\alpha]$ is the expected value.

As an example, consider the tuple

$$t = (\text{Centurion, AB33, [March 1} \sim \text{March 31} - \text{June 1} \sim \text{June 30]}).$$

For this tuple, containing a period timestamp, the four credibility values yield the following results:

$$Replace(\texttt{INDETERMINATE},\ t) = t$$

$$Replace(\texttt{EXPECTED},\ t) = (\text{Centurion, AB33, [March 15} - \text{June 15]})$$

$$Replace(\texttt{MAX},\ t) = (\text{Centurion, AB33, [March 31} - \text{June 1]})$$

$$Replace(\texttt{MIN},\ t) = (\text{Centurion, AB33, [March 1} - \text{June 30]})$$

Using the *Replace* function, we are in a position to define the meaning of the from clause in indeterminate semantics, including specification of how the default credibility can be overridden for a particular correlation variable.

$$\begin{aligned} &[\![\langle from\ list\rangle]\!]_{ind}(\delta, d) \\ &= [\![\langle from_1\rangle,\ \ldots,\ \langle from_n\rangle]\!]_{ind}(\delta, d) \\ &= [\![\langle from_1\rangle]\!]_{ind}(\delta, d) \times \ \ldots \ \times [\![\langle from_n\rangle]\!]_{ind}(\delta, d) \end{aligned}$$

where each of the $\langle from_i\rangle$s can be either of two constructs:

$$[\![\langle relation\rangle]\!]_{ind}(\delta, d) = \{t' \mid t \in \langle relation\rangle \wedge t' = Replace(\delta, t)\}$$

$$[\![\langle relation\rangle \texttt{ WITH CREDIBILITY } \delta']\!]_{ind}(\delta, d) = \{t' \mid t \in \langle relation\rangle \wedge t' = Replace(\delta', t)\}$$

When $\langle from\ list\rangle$ is given a credibility of $\delta = \texttt{INDETERMINATE}$, it retains all of the indeterminacy present in the temporal values of d, not altering them at all. Hence,

$$[\![\langle from\ list\rangle]\!]_{ind}(\texttt{INDETERMINATE}, d) = [\![\langle from\ list\rangle]\!]_{SQL}(d).$$

5.7 Result of the Example Query

At this point, the indeterminate semantics of the select statement has been specified. As an example, we provide tuple calculus semantics for the query given in Figure 3.

$$\begin{aligned} [\![Q]\!]_{ind}(d) = \{(r.Warehouse,\ r.Lot_Num,\ p.SerialNum,\ r.When) \mid \\ r \in Replace(\texttt{INDETERMINATE}, Received) \\ \wedge\ p \in Replace(\texttt{INDETERMINATE}, In_Production) \\ \wedge\ p.Model = \texttt{'Centurion'} \wedge r.Part = \texttt{'wingstrut'} \\ \wedge\ True \in \{Before_I(p.During_{starting},\ r.When,\ 60) \cap \\ Before_I(r.When,\ p.During_{terminating},\ 60) \end{aligned}$$

Here the Cartesian product and projection operators have been expressed in tuple calculus.

If this query is applied to the database given in Figure 1, it will result in three tuples, shown in Figure 4. First, the time values in the underlying relations are unchanged because the query uses a correlation credibility of `INDETERMINATE`. The where clause eliminates every tuple from In_Production except the Centurions. Likewise, the where clause also eliminates every tuple from $In_Production$ except the wing strut tuples.

The shipment of lot number 23 was definitely received during production of Centurion serial number AB33; it satisfies the overlap with every plausibility. The other shipments might have been received. Lot number 30 satisfies the overlap for plausibilities lower than 60 because (May 30 $\sim$ June 18, uniform) is before (June 1 $\sim$ June 30, uniform) for every ordering plausibility below 65. The other shipment, however, arrived too late in June to be considered plausible. It is plausible that lot number 31 arrived before the end of production for ordering plausibilities of 28 or less only. For production of the Centurion serial number AB34, all the shipments arrived too early, except for lot number 31 from the Cessna warehouse.

5.8 Correctness

The intuitive semantics ($[\![]\!]_{ind}$), motivated in Section 5.3, applies the predicate to each completion and thus is impractical, whereas operational semantics ($[\![]\!]_{op}$) can be efficiently implemented. The correctness of operational semantics hinges on two requirements. The first is that the two semantics agree where the intuitive semantics is defined, that is, for the possible and definite interpretations. The possible interpretation has an ordering plausibility of $\gamma = 1$; the definite interpretation has $\gamma = 100$. The interpretations concern ordering plausibility only ; they apply at any correlation credibility.

THEOREM 3. *$[\![]\!]_{ind}(\gamma, r)$ and $[\![]\!]_{op}(\gamma, r)$ are equivalent for $\gamma = 1$ and $\gamma = 100$.*

The second requirement is that the operational semantics be monotonic: as the plausibility increases, the result must move from possible interpretation towards definite interpretation. This ensures that the semantics at intermediate plausibilities is consistent with that of the possible interpretation.

THEOREM 4. *$[\![S]\!]_{ind}(\delta, \gamma, r)$ is monotonic in γ.*

6. IMPLEMENTATION OVERVIEW

Changes to the semantics to support valid-time indeterminacy induce changes in implementation. These changes are isolated to the representation of instants, intervals, and periods, and to the new or modified temporal operators such as $Before_I$ and *Replace*. In the next two sections, we describe

the data structures and algorithms to implement these new or modified operators. Our goal is to provide support for valid-time indeterminacy without adversely impacting storage requirements or query evaluation efficiency.

At first glance, support for valid-time indeterminacy appears to be expensive. Some of the modified operators, e.g., $Before_I$, are executed in the "inner loop" of query processing, potentially performed many times for each combination of tuples in the queried relations. Significant slowdown of these operators would have a dramatic effect on the overall speed of query evaluation. We show below how the new operators can be implemented efficiently.

7. INDETERMINATE TIMESTAMP FORMATS

Valid-time indeterminate instants, periods, and intervals model new kinds of temporal information. To represent indeterminate temporal values, new temporal data types, or *timestamps* [Jensen et al. 1994], are needed. In this section, we briefly describe indeterminate timestamps. We present indeterminate instant timestamps only; the interval and period timestamps are natural extensions of the instant timestamps [Dyreson and Snodgrass 1995a].

The instant format described here builds upon the determinate instant format; a full description of this format is given elsewhere [Dyreson and Snodgrass 1995a]. A determinate instant timestamp combines a type tag (to indicate the kind of instant, e.g., determinate or special, such as 'now' [Clifford et al. 1997]) with a signed integer representing a distance (in chronons, or more precisely, in *granules*[7]) from the timeline *origin* or *anchor-point*. The type tag occupies three bits and the signed integer 29, 61, or 93 bits, depending on the maximum range and granularity of the timestamp. The 64-bit timestamps can store a range of historical times to the granularity of a microsecond, or times within a range of 36 billion years (all of time, back to the big bang) to the granularity of a second. It is important to point out that the range and granularity of an instant timestamp are stored in the schema rather than in the timestamp.

Indeterminate instants cannot use the determinate timestamp because an indeterminate instant is more than a single time: it is two times and a probability mass function. To represent indeterminate instants, we add four new formats. These four formats are a combination of *compact* or *general* with *standard* or *nonstandard* distributions. The combinations are explained in detail below.

Each indeterminate timestamp format has the three basic parts needed to describe an indeterminate instant: a lower support, an upper support, and a probability mass function. These three parts are encoded in the

[7]A granule is a coarse-grained grouping of chronons, e.g., chronons can be grouped into days, years, or months. The timestamp stores the distance in terms of granules so that large ranges of time, e.g., 10 million years, can be stored compactly by counting in coarse granules, e.g., in millennia.

Table II. Encodings in Timestamp Formats

Format	Starting Time	Terminating Time	Probability Distribution
determinate	explicit	implicit	implicit
compact, standard	explicit	implicit	implicit
compact, nonstandard	explicit	implicit	explicit
general standard	explicit	explicit	implicit
general, nonstandard	explicit	explicit	explicit

timestamp either implicitly or explicitly. Table II indicates for each format whether the representation is explicit or implicit. For example, the determinate format has an explicit lower support, but an implicit upper support (identical to the lower support) and an implicit probability mass function (the mass function is *missing*).

Compact indeterminate formats implicitly encode the upper support. Implicit encoding consists of a *chunk-size* and a number of *chunks*. The upper support is computed by adding the number of chunks, each of size chunk-size, to the lower support. For example, to represent a period of indeterminacy of seven hours using chunks, the timestamp would record that there are seven hour-sized chunks. The chunking scheme was developed to meet the expectation that regular periods of indeterminacy, e.g., N hours, N days, or N years, will be the norm. If the format is not compact, it is said to be *general* and the upper support is stored explicitly as a count of granules from the anchor point.

If the chunk-size and number of chunks can be stored in two small fields, then chunking is a very efficient method of encoding a terminating time. In our formats, the chunk-size is a four-bit field and the number of chunks a seven-bit field, or eleven bits in toto. Since the chunk-size is four bits, only a limited number of chunk-sizes are available. One of the duties of the database implementor is to specify chunk-size tables, one for each supported granularity. Common chunk-sizes are seconds, hours, days, and weeks. The space efficiency of chunking comes at the expense of some run-time computation, since the terminating time must be computed on the fly. The computation costs one addition to add the chunks to the lower support.

The timestamp representation of a probability mass function is the name of a mass function (a 16-bit identifier). Only the name of the mass function, for example, uniform, normal, etc., is stored with the timestamp; the actual mass function is stored separately, as described in Section 8.1.2. Instants with a uniform distribution or a distribution that is missing are termed *standard* distributions. Since these distributions will likely be common, we optimized their representation as a single bit (to toggle between uniform and missing) rather than a 16-bit identifier.

The user specifies the kind of timestamp, compact or general, and the kind of distribution, standard or nonstandard, to use when defining or altering a temporal attribute, as discussed in Section 4. The design of the indeterminate timestamp formats optimizes representation of the common

```
function Before_I(in α, β : instant; in γ : integer) : set of boolean;
    begin
        if α is β then return {True}
    /*Are the periods of indeterminacy disjoint?*/
        else if α* ≤ β_* then return {True}
        else if β* < α_* then return {False}
    /*The periods of indeterminacy must overlap.*/
        else if γ = 1 return {True, False}
        else if (γ = 100
                ∨ β has a missing mass function
                ∨ α has a missing mass function) return {}
        else return PROB_α_LEQ_β(α, β, γ)
    end;
```

Fig. 6. Interface to $Before_I$.

mass functions (standard mass functions cost only a single bit). The chunking scheme and the use of standard distributions yield a compact timestamp. SQL-92's limited `TIMESTAMP` format without fractional seconds and without indeterminacy (assuming that the *positions* in the SQL-92 timestamp are four-bit nibbles) is 56 bits. Our indeterminate compact timestamp with the same range and granularity as the SQL-92 datetime format requires only 64 bits. The smallest indeterminate timestamp is just 32 bits (a compact, standard format with a range of 2^{18} granules and a chunked period of indeterminacy). The largest is 208 bits (a general, nonstandard format with an upper and lower support within 2^{93} granules of the granularity anchor point).

8. IMPLEMENTING OPERATORS

In this section, we discuss implementing $Before_I$ and *Replace*.

8.1 Implementing $Before_I$

We observed in Section 5 that the semantics of temporal constructors and predicates such as `OVERLAPS` and `PERIOD` are ultimately based on $Before_I$. If the instants being compared by $Before_I$ are determinate, then $Before_I$ is the " $\leq$" relation on the domain of time values (integers extended with special values representing the beginning and end of time). Indeterminate instants complicate the implementation of $Before_I$. In the indeterminate semantics, it may be necessary to compute the probability that one instant is before another—a potentially costly computation. We show below how this computation can be made efficient.

8.1.1 *The Common Interface.* The interface to the $Before_I$ routine is given in Figure 6. The interface determines if the relatively costly compu-

tation of the ordering probability can be avoided. If α and β are the *same* instant, then $Before_I$ is trivially true, since an instant is always equal to itself. We test for 'sameness' by tagging each instant in memory and checking the tags. $Before_I$ is also trivial if α's and β's indeterminacy periods are disjoint. Disjointness implies that one instant is before the other in all possible cases. We anticipate that disjoint periods of indeterminacy will be common. Even if periods of indeterminacy overlap, there are several special cases. If the periods of indeterminacy overlap, then there is at least a small probability that each instant is less than or equal to the other. Consequently, the relationship is satisfied for a plausibility of 1 (i.e., any non-zero probability), but cannot be satisfied for a plausibility of 100 (i.e., a probability of 1.0). For plausibilities between 1 and 100, if either mass function is missing, then no relationship between the instants can be determined. If no special case applies, then the ordering probability, $\mathbf{Pr}'[\alpha \leq \beta]$, must be calculated.

8.1.2 *Probability Mass Function Representation.* In this section, we describe a data structure to store a probability mass function. We present the data structure first since it impacts the algorithm design.

In general, a function can either be computed on the fly or precomputed and its values cached, say, in an array. The latter strategy is best for a probability mass function. $Before_I$ is executed in the "inner loop" of query processing, performed many times during a query. We anticipate that many useful probability mass functions are not easily computable functions, making computing values on the fly expensive in terms of execution time whereas table-lookup is quite cheap, although potentially expensive in terms of space.

To attain reasonable storage costs, the probability mass function is *approximated*. We approximate a mass function as follows. First, the mass is *quantized*; that is, it is parceled into indivisible, discrete chunks of probability. The quanta can be thought of as *rods* of equal mass but (possibly) differing lengths. If a probability mass function has P rods in total, then the mass of each rod is $1/P$. The number of rods is called the *precision* of the approximation. Next, the mass function is sampled at C evenly-spaced points. C is called the *coarseness of approximation.* For simplicity, we assume that the domain of the mass function is [0,1] (i.e., the domain is normalized). The sample points are $\{1/2C, 3/2C, \ldots, (2C - 1)/2C\}$. The coarseness would usually be much larger than precision. (For instance, in our experiments we use a coarseness of 2^{16}, but a precision of 2^{8}.) Finally, the rods to the left and right of each sample point are counted and recorded. The count of rods to the left and right of each sample point is the approximated mass function. The rod covering a point is not counted (this is the error in approximation).

The approximation of the uniform mass function with a coarseness of 8 and a precision of 3 is shown in Figure 7, where the sample points are 1/16, 3/16, . . . , 15/16. The position of the rods covering the sample points indicates approximately how much probability is to the left and right

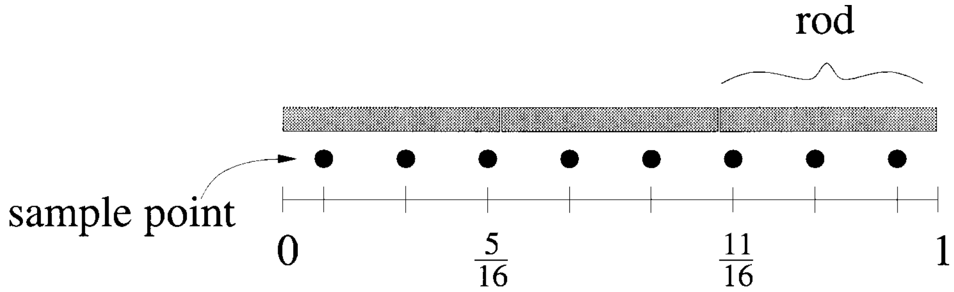


Fig. 7. The approximated uniform mass function with $P = 3$ and $C = 8$.

of a sample point. From left to right in the figure, the first rod covers the first three sample points. At each sample point under the first rod, there are no rods to the left and two rods to the right, so most of the probability, 2/3 of the total, is located to the right. The second rod covers the next two sample points. For each of these sample points, 1/3 of the total, is located to the right. The second rod covers the next two sample points. For each of these sample points, $1/3$ of the total mass is to the left and 1/3 is to the right of the point. The final rod covers the last three sample points, each with 2/3 of the total mass to the left. Note that in the approximation, the mass to the left and right of a sample point is within 1/3 of the actual mass in the unapproximated distribution.

The rod and point method of approximating a probability mass function has some limitations. Coarseness and precision restrict the variety of functions that can be approximately represented. If coarseness equals precision, then only uniform probability mass function (every point is equally likely) can be represented (a different rod on every point). As coarseness and precision diverge, more mass functions can be represented. In general, with a precision of P and a coarseness of C, at most $\binom{C}{P}$ different probability mass functions are possible. Further, "spiky" mass functions cannot be approximated. That is, mass functions that have a mass of more than $1/P$ spread over less than $1/C$ of their domain cannot be approximated (which means that two or more rods would have to span the same point). It is the database implementor's task to choose the appropriate C and P values to support the kinds of mass functions that are of interest to users.

Using the rod and point method, a probability mass function is approximated with an absolute error of less than $1/P$. That is, the probability of a possible instant in the approximated distribution is within $1/P$ of the actual probability. If the difference between the probabilities is more than $1/P$, then the approximation has been done incorrectly, as a new rod should have been introduced.

The approximated mass function is stored in a binary tree rather than in an array. There is one leaf for each sample point. For instance, the first leaf

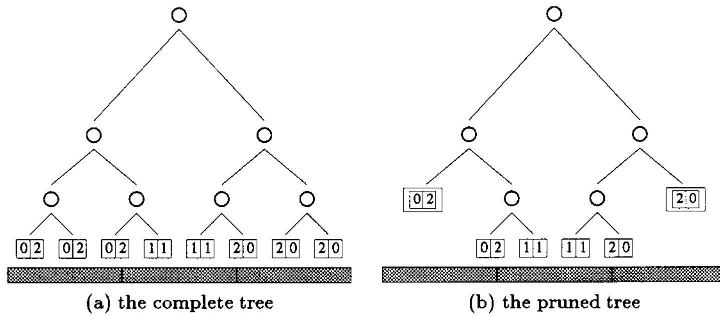


Fig. 8. The tree for the approximated uniform mass function with $P = 3$ and $C = 8$.

in a preorder traversal corresponds to the sample point $1/C$. At each leaf in the tree, the number of rods to the left and right of the sample point are stored. For example, in the approximation of the uniform mass function shown in Figure 7, there are no rods to the left and two rods to the right of the first point. The example shows that the number of rods left and right of a sample point often sum to $P - 1$, since the rod covering the node is uncounted. The tree for the approximated uniform mass function is shown in Figure 8(a).

In the tree shown in Figure 8, C and P are small values, consequently the entire tree can be easily stored in just a few bytes. But when C and P are large, it is infeasible to store the full tree, nor do we need to do so. We are primarily interested in recording where each rod ends. Observe that if both children have the same count, then no rod ends within the subtree rooted at the parent (and all nodes in the subtree will have the same count). All such subtrees can be pruned, keeping only the root of the subtree, which is specially marked. When traversing a pruned subtree, the tree traversal algorithm treats a specially marked node as the root of a "virtual" subtree and traverses the subtree as though it were stored. The pruned tree for the example distribution is shown in Figure 8(b), where the specially marked nodes are represented as a box within a box.

The tree pruning technique saves quite a bit of space. The pruned tree has at most $2P$ leaves (one leaf might be needed per rod end) and could have as few as P leaves. In contrast, the unpruned tree has C leaves (in general $C \gg P$). The number of interior nodes also varies, with as few as $P - 1$ interior nodes and as many as $2P - 1$ interior nodes in a tree. Each interior node uses two $\log_2(C)$-bit pointers while each leaf node uses two $\log_2(P)$-bit fields to store the number of rods. For $C = 2^{16}$ and $P = 2^8$, the storage cost of a pruned search tree is between 1.5K and 3K bytes.

The distribution tree efficiently stores the approximated probability mass function, but the approximation impacts the computation of $\mathbf{Pr}'[\alpha \leq \beta]$, changing the problem to one of rod counting.

8.1.3 *An Overview of Computing* $\mathbf{Pr'}[\alpha \leq \beta]$. Calculating the probability that one instant is before another using the approximated mass function can be reformulated as a rod–counting problem. Assume that there are two rows of P rods. The two rows, which we call the α-row and the β-row, are parallel to each other, as shown in Figure 9. Note in the figure that the length of each rod can vary. (The rods in the figure have the same area to indicate that short, fat rods have the same mass as long, skinny rods.) The rod-counting problem is to count the pairs of rods, one rod from each row, such that the rod from the α-row is before the rod from the β-row. Each such pair represents a contribution of $1/P^2$ to $\mathbf{Pr'}[\alpha \leq \beta]$.

The rod–counting problem is complicated by the fact that several rods may be located within a single chronon, so some rods in β could be strictly before those in α, yet may still contribute to $\mathbf{Pr'}[\alpha \leq \beta]$. For the purpose of computing $\mathbf{Pr'}[\alpha \leq \beta]$, each chronon has an indivisible mass; that is, all the rods entirely within the same chronon should be treated as a single rod with a mass equivalent to the total mass of the constituent rods. For example, consider an indeterminate instant with a uniform mass function and a set of possible chronons consisting of only two chronons. There are $\lfloor P/2 \rfloor$ rods within each chronon, consequently each chronon in this indeterminate instant has an indivisible mass of 0.5.

The rod–counting problem also differs from the original problem of computing the probability that one instant is before another in a subtle, but significant, way: the sum of the mass in pairs of rods where α's rod is before β's rod is not quite the same as $\mathbf{Pr'}[\alpha \leq \beta]$. Consider a pair of rods, neither of which is before the other (the rods are at the same place in the overall ordering of rods). Each rod represents the probability that the instant is located during a certain range of chronons, but how the probability is distributed among the chronons within that range is unknown. Although neither rod is before the other, it is probably the case that some chronon within the range represented by the rod is before a chronon in the range represented by the other rod. The rod–counting problem does not count the small probability ($\leq [1/P^2]$) of this case, and thus undercounts $\mathbf{Pr'}[\alpha \leq \beta]$. Below, we quantify the error on the rod-counting technique.

The algorithm for counting pairs of rods is based on a divide-and-conquer technique. Each step of the algorithm is illustrated in Figure 9. The first step is to choose a *pivot*. A pivot is a rod in α's row of rods. The pivot splits the rods in α-row into three groups: those before the pivot, α_{before}; those after the pivot, α_{after}; and the pivot itself.

The second step is to identify where the right-end of the pivot belongs in the ordering of β's rods.The right-end of the pivot divides β's row of rods into three parts: those before the right-end of the pivot, β_{before}; those after the right-end, β_{after}; and, perhaps, a rod that overlaps the right-end, $\beta_{overlap}$.

The third step is the conquer step. Observe that all the rods in $\alpha_{before} \cup$ *pivot* are before all the rods in β_{after}. Each pair of rods, one chosen from each of these two groups, adds $1/P^2$ to a running sum of $\mathbf{Pr'}[\alpha \leq \beta]$. If the

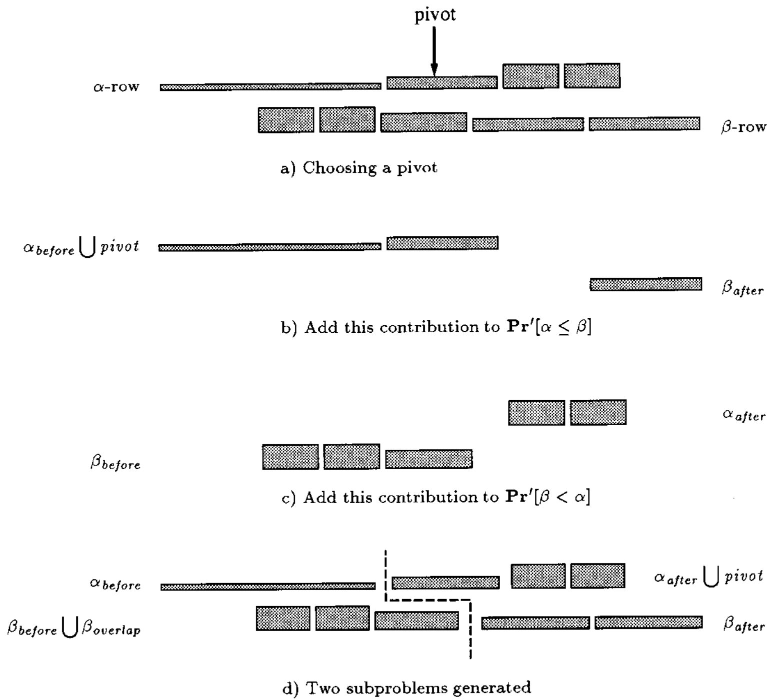


Fig. 9. A rod–counting operation.

running sum (scaled by 100) exceeds the plausibility, γ, then the algorithm terminates, since the plausibility has been met and $Before_I$ is {*True*}. This is called an "early exit" condition.

Similarly, all the rods in β_{before} are before the rods in α_{after}. Each pair of rods, one chosen from each of these two groups, adds $1/P^2$ to a running sum of $\mathbf{Pr}'[\beta < \alpha]$. If the running sum (scaled by 100) exceeds $100-\gamma$, then the algorithm terminates, since $Before_I$ is {*False*}. This is the only other "early exit" condition.

If an early exit is not taken, then two subproblems remain to be solved. The algorithm has yet to determine the relationships between the rods in α_{before} and those in $\beta_{before} \cup \beta_{overlap}$, as well as the relationships between the rods in $\alpha_{after} \cup pivot$ and those in β_{after}. Each of these subproblems is solved recursively in the next "round" of the algorithm.

8.1.4 *Choosing the Pivot.* The choice of pivot is an important factor in controlling the algorithm. The algorithm chooses as the pivot the rod corresponding to half of the remaining rods in α (those rods that have yet to be counted). This choice enables the algorithm to reach an "early" exit condition quickly. Overall, the total work performed by the algorithm is to count all P^2 pairs of rods. But the counting can stop when enough pairs are counted to determine if either $\mathbf{Pr}'[\alpha \leq \beta]$ or $\mathbf{Pr}'[\beta < \alpha]$ is satisfied (the

early exit conditions). It is better if, in the first few pivot choices, an algorithm maximizes the pairs of rods it counts, since it will then hit an exit condition in fewer pivots.

THEOREM 5. *The kth pivot will count $P^2/2^{\lfloor \log_2(k) \rfloor + 1}$ pairs.*

For example, for a precision of 2^8, after the 14th pivot, the algorithm will have counted 93% of the total number of pairs. In other words, to approximate the ordering probability to within 10%, at most fourteen pivots must be performed.

As we pointed out earlier, the approximation by rods and points leads to an undercounting of $\mathbf{Pr}'[\alpha \leq \beta]$. However, this undercounting is small.

THEOREM 6. *The undercount is less than $2/P$.*

8.1.5 *Implementation Details for Computing* $\mathbf{Pr}'[\alpha \leq \beta]$. The code for the "pivoting" algorithm is shown in Figure 10. The counting stops when the count of pairs exceeds the needed number of true pairs or false pairs (it simultaneously solves for both $\mathbf{Pr}'[\alpha \leq \beta]$ and $\mathbf{Pr}'[\beta < \alpha]$), or when all the possible pivots have been tried (an undercount has occurred, and we assume that $\mathbf{Pr}'[\alpha \leq \beta]$ is false). The most important feature of this code is that the majority of instructions are "cheap" integer operations: shifts, assignments, and additions. There are only two multiplications, no divisions, and no floating point operations. Although, for pedagogical reasons, we have presented the pivoting code as a recursive procedure, the procedure is implemented using a queue and iteration, thus avoiding the expense of recursive procedure calls and supporting breadth-first recursion. One final observation: calculating the number of true and false pairs has been reduced to a table-lookup, since the ordering plausibility, γ, can take on only 100 different values.

8.1.6 *Other Comparison Operators*. Implementation of other primitive comparison operations, such as $\mathbf{Pr}'[\alpha = \beta]$ to support $Equals_I$ and probabilistic $\mathbf{Pr}'[\alpha < \beta]$ to support $Strictly_Before_I$, varies only marginally from the implementation of $\mathbf{Pr}'[\alpha \leq \beta]$; the pivoting technique is a general strategy. We do not consider these operations further in this paper, other than to note that, on average, fewer pivots are needed to compute $\mathbf{Pr}'[\alpha = \beta]$ than $\mathbf{Pr}'[\alpha \leq \beta]$, since the probability that two instants are equal is very small (for large periods of indeterminacy). Consequently, it is a relatively efficient operation, which is why in our experiments we focus on the more expensive $\mathbf{Pr}'[\alpha \leq \beta]$.

8.2 The *Replace* Function

The *Replace* function changes an indeterminate time value in one of four ways. It could replace the value with the lower support, replace it with the upper support, replace it with the expected value, or leave the value unchanged. The straightforward pseudocode for *Replace* for an instant is given in Figure 11. The code for starting and terminating period time

```
function PROB_α_LEQ_β(in α, β : instant; in γ : integer) : set of boolean;
  const
    P : integer = 256; C : integer = 65536;
    α_tree, β_tree : probability_mass_function_tree(C,P);
    plausibility_map : array[1..100] of 1..⌊P²/100⌋;
  var
    false_pairs, true_pairs, pivot, α_mid : integer;
    leaf : tree_node_pointer;
  procedure ROD_COUNTING(in α_from, α_to, β_from, β_to : integer);
    var
      pivot, α_mid, α_before, α_after, β_before, β_after : integer;
    begin
    /* Check the exit conditions */
      if (true_pairs ≤ 0) ∨ (false_pairs < 0) then return;
      if ((α_to − α_from) = 0) ∨ ((β_to − β_from) = 0) then return;
    /* Calculate pivot */
      pivot ← α_from + ((α_to − α_from) div 2);
    /* Figure out α's contribution */
      α_before ← pivot − α_from;
      α_after ← α_to − pivot;
    /* Figure out the chronon in which the pivot ends using binary search */
      α_mid ← binary_search(α_tree, pivot);
    /* Find the rod in β just after the pivot's chronon using binary search */
      leaf ← binary_search(β_tree, α_mid);
    /* Figure out β's contribution */
      β_before ← leaf.left_rods − β_from;
      β_after ← β_to − leaf.right_rods;
    /* How much is the total contribution? */
      true_pairs ← true_pairs − ((α_before + 1) × β_after);
      false_pairs ← false_pairs − (α_after × β_before);
    /* Continue counting */
      ROD_COUNTING(α_from, α_from + α_before, β_from, β_from + β_before);
      ROD_COUNTING(α_to − α_after, α_to, β_to − β_after, β_to);
    end;
  begin
    true_pairs ← plausibility_map[γ];
    false_pairs ← P² − true_pairs;
    ROD_COUNTING(1, P, 1, P);
    if (true_pairs ≤ 0) return {True}
    else if (false_pairs ≤ 0) return {False}
    else return {};
  end;
```

Fig. 10. The pivoting algorithm.

values, as well as for intervals, is similar, and omitted for brevity. In this pseudocode we assume that the expected value for the probability mass

```
function ReplaceInstant(in δ : credibility; in α : instant) : instant;
    const
        α_tree : probability_mass_function_tree(C, P);
    var
        j : integer;
    begin
        case δ
            INDETERMINATE: return α;
            MAX: return α*;
            MIN: return α_*;
            EXPECTED: return α_* + (α_expected × (α* − α_*))
    end; { ReplaceInstant }
```

Fig. 11. The *Replace Instant* algorithm.

function has been precomputed for the normal interval [0,1]. This value is cached and used by the *Replace* function to compute the expected value for a nonnormal interval, that is, a period of indeterminacy.

8.3 Impact of Indeterminacy on the Determinate Implementation

In parallel with the theorem of reducibility given in Section 5.8, conventional SQL queries on determinate relations incur no additional execution overhead under the new semantics, and executing such queries on indeterminate relations adds little overhead, since $Before_I$'s for a plausibility of 100 are very efficient.

9. EMPIRICAL ANALYSIS OF THE IMPLEMENTATION

We implemented the indeterminate operations in a prototype system for temporal support called MULTICAL, which is written in the C programming language [Soo et al. 1992].[8] We compiled the code using the GNU C compiler, version 2.7.2, with compiler optimization fully enabled. We used a precision of 2^8 and a coarseness of 2^{16} in the code for $Before_I$. These values limit the maximum error in the pivoting algorithm to less than 1%. We also implemented $Before_I$ with a maximum possible error of 10%. This version of $Before_I$ performs (at most) 14 pivots, as discussed in Section 8.1.4.

We tested the performance of each operation in isolation first. All tests were performed on a DEC-Alpha 3000 Model 400 (a 133.33 MHz machine). The timings for each test were collected using the `atom` tool [Srivastava and Eustace 1994], which allowed us to count machine cycles. Table III shows the results for each operation. The execution times shown in the

[8] C code for the operations discussed in the previous section, as well as for all the experiments discussed here, is available via the WWW at `http://www.cs.jcu.edu.au/~curtis/htmls/indeterminacy.html`.

Table III. Timings on Indeterminate Operations (in machine cycles)

Operation	Determinate Cost	Indeterminate Best-Case Cost	Indeterminate Worst-Case Cost
Before (Determinate)	73	NA	NA
$Before_I$ - 1% error	NA	77	86930
$Before_I$ - 10% error	NA	77	4960
Replace	NA	33	239

table only include the cycles actually spent within the function and exclude the cost of the function call itself. "NA" denotes "not applicable." The *Replace* and determinate *Before* operations are relatively cheap. The worst case for *Replace* occurs with the EXPECTED credibility because it involves floating point multiplication. The best– and worst-case behaviors of $Before_I$ vary significantly.

To further examine $Before_I$'s behavior, we devised several additional tests, designed to capture both the worst-case and the expected-case performance of the pivoting algorithm. The worst case for $Before_I$ happens when the two indeterminate instants span the same chronons and have uniform distributions. We tested this worst-case performance of $Before_I$ on a pair of instants, each of which has a period of indeterminacy of one million chronons. The results are given in Figure 12. The graph plots the execution time (in machine cycles) of $Before_I$ for the plausibility values 1 to 100. As the ordering plausibility approaches 50, the execution times increase because more pivots are needed to determine the outcome of $Before_I$. The average worst-case $Before_I$ operation with 1% error across all plausibilities is approximately 2781 machine cycles, with a high of 86930 machine cycles at a plausibility of 50 and a low of 77 machine cycles. With a maximum error of 10%, the average worst case for $Before_I$ is somewhat less, 1246 machine cycles, with a high of 4960 machine cycles at a plausibility of 50.

The worst-case performance does not always occur at a plausibility of 50, but depends on the relative positions and mass functions of a pair of indeterminate instants. Two instants with uniform distributions which have partially (but not fully) overlapping periods of indeterminacy will exhibit worst-case performance at plausibilities other than 50. Using the same two instants from the first test, we tested a range of relationships between their periods of indeterminacy, from no overlap to complete overlap. We fixed the position of one instant in chronon space and slid the other instant relative to the fixed instant. Figure 13 shows the results. The z-axis is the cost (in machine cycles) of a single call to $Before_I$ (we used the 10% error version). The x-axis is the plausibility. The y-axis is the relative position of the two instants, that is, "far apart" indicates no overlap in the periods of indeterminacy whereas "even" means complete overlap. The figure shows that if the instants do not overlap (a common case), $Before_I$ is very cheap. If the instants overlap, $Before_I$ only exhibits poor performance

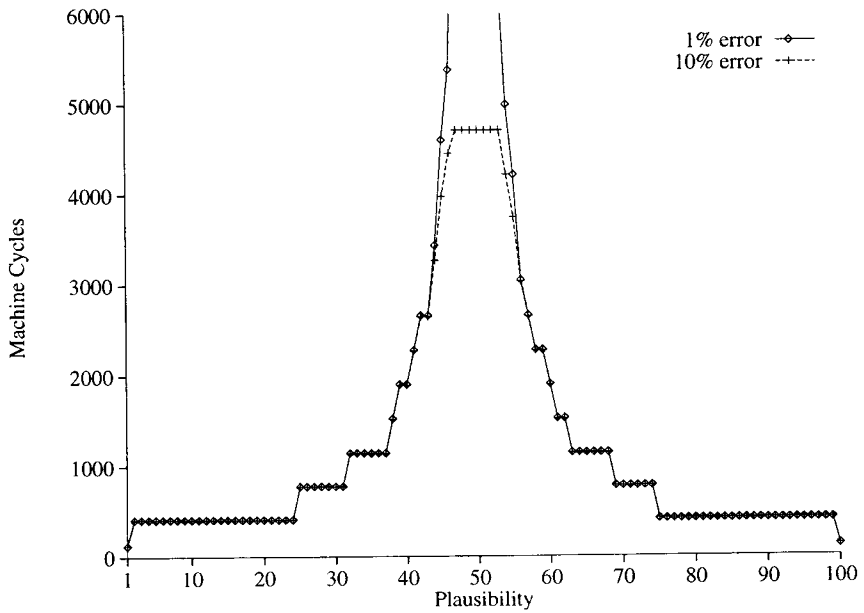


Fig. 12. worst-case performance of $Before_I$.

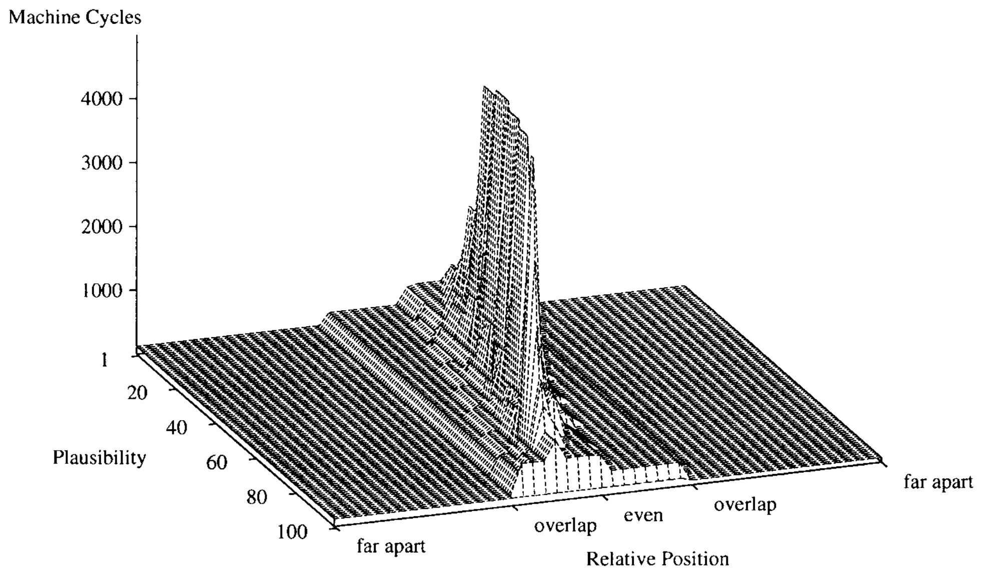


Fig. 13. Sliding one instant relative to another performance of $Before_I$.

along a central ridge. Note that the graph in Figure 12 is a slice of the graph in Figure 13 at the "even" point along the "relative position" axis.

While examining worst-case behavior is sometimes illuminating, we anticipate that it will be uncommon for two instants to have overlapping

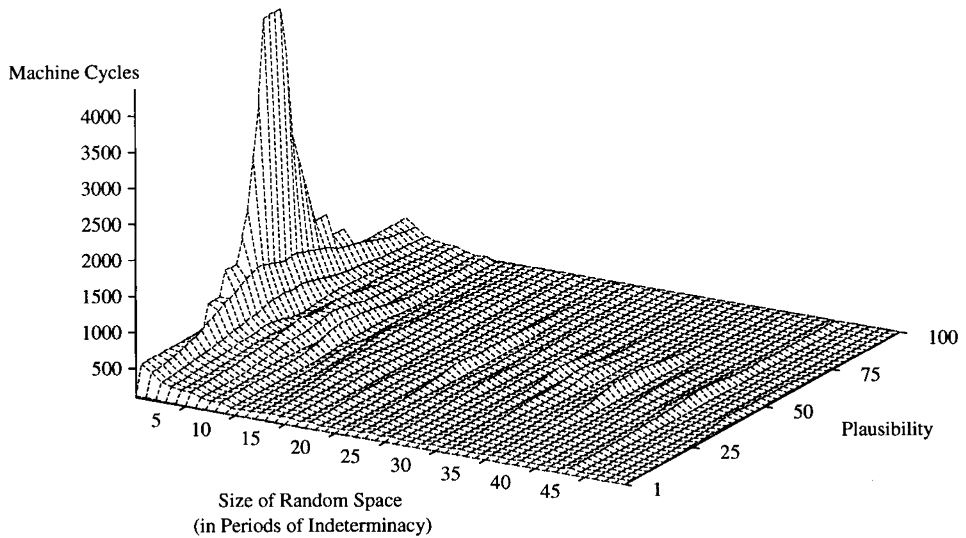


Fig. 14. The cost of comparing ten instants randomly placed in a chronon space of varying size.

periods of indeterminacy, and that worst-case behavior will be rarer still. For example, consider a relation of employee hires that has an instant timestamp column that records the day that the employee was hired. The day an employee was hired is an indeterminate instant, assuming a common timestamp granularity of a second. Suppose we query this relation to determine which employees were hired before the third fiscal quarter began. The quarter began at 8 AM on October 1st. It is unlikely that most of the hiring instants overlap 8 AM on October 1st. Hence, the $Before_I$ comparison for most of the instants in the relation will be very efficient, and the impact of the other comparisons on the total work done in the query will be slight.

To explore this aspect further, we devised a further test. We randomly placed ten instants, each of which had a one-day period of indeterminacy (86,400 chronons) and a uniform distribution, in a chronon space that varied between 1 and 50 days (between 86,400 and 4,320,000 chronons) in size. For every plausibility between 1 and 100, we tested $Before_I$ on every possible combination of instants (10^2 possible combinations) at plausibilities ranging from 1 to 100. Instants were not compared to themselves. We rerandomized the location of the instants between each test (i.e., per one hundred comparisons). The results are depicted in Figure 14. The graph in Figure 12 is a slice of this graph at a value of one unit of random space. The graph shows that in a normal mix of instants, rare worst-case situations have little impact on overall performance. Only when the size of the random space is small is the cost of $Before_I$ significant.

To this point, we have not determined how much more expensive $Before_I$ will be than *Before*. To measure the relative cost of $Before_I$, we reran the "random placement of instants" test described above on both *Before* and

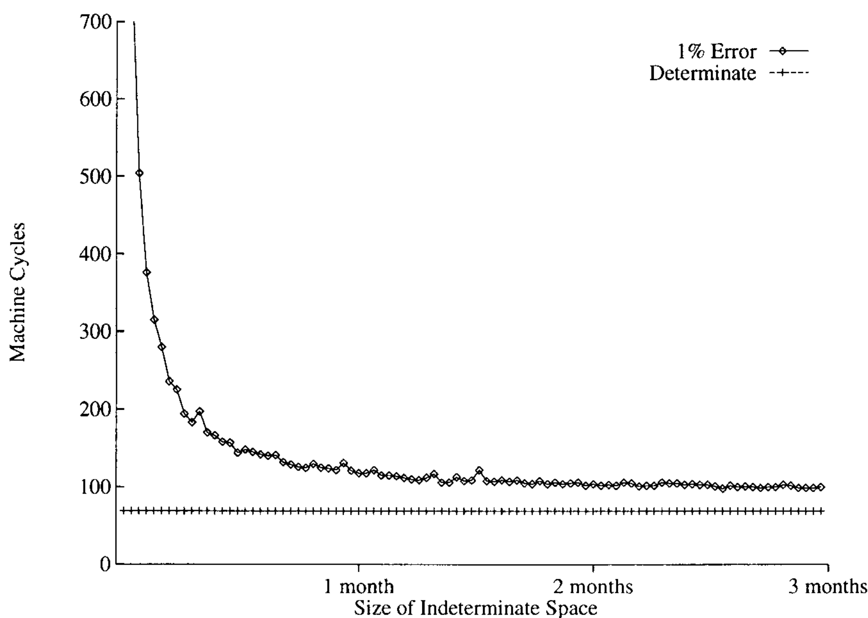


Fig. 15. The average cost of comparing ten instants randomly placed in a chronon space of varying size.

$Before_I$. But this time we let the size of the random chronon space vary between 1 and 93 days (3 months) rather than 1 and 50. For each day, we averaged the cost of the $Before_I$ operation across all the plausibilities, 1 to 100. The results are plotted in Figure 15. When all ten instants are randomly placed in a chronon space three months in size (i.e., there are ten employee hires in three months and only these hires are used in the query), $Before_I$ is approximately one and one-half times as expensive as *Before*. This difference remains approximately the same as we further increase the size of the random space.

Although the run-time cost of each operation considered in isolation is informative, it does not address the "actual" cost of a query with indeterminate information, since the frequency of operations and the interactions between operations are absent from the analysis. In addition, these operations are only one portion of query evaluation; many other operations are performed in most queries. To measure $Before_I$ and the other operations in context, we designed a test of a complete query. We hand-compiled the example SQL query given in Figure 3 into the series of MultiCal calls shown in Figure 16. The determinate and indeterminate sequences of calls differ only slightly; the differences are highlighted in italics in the indeterminate sequence. We used the compact indeterminate timestamp formats, which are more expensive to unpack, but have the same space cost as the determinate timestamps used in the experiment. Note that each sequence does $O(n^2)$ unpacks, that is, all possible combinations of instants and

```
{ The determinate sequence }
    for every combination of r and p
        unpack_instant(r.When);
        unpack_period(p.During);
        c1 ← Before(r.When, p.During_to);
        c2 ← Before(p.During_from, r.When);
        if (c1 and c2) then
            add temp to result
        end
    end
```

```
{ The indeterminate sequence }
    for every combination of r and p
        unpack_instant(r.When);
        unpack_period(p.During);
        c1 ← Before_I (r.When, p.During_to, 60);
        c2 ← Before_I (p.During_from, r.When, 60);
        if (c1 and c2) then
            add temp to result
        end
    end
```

Fig. 16. MULTICAL calls for example query.

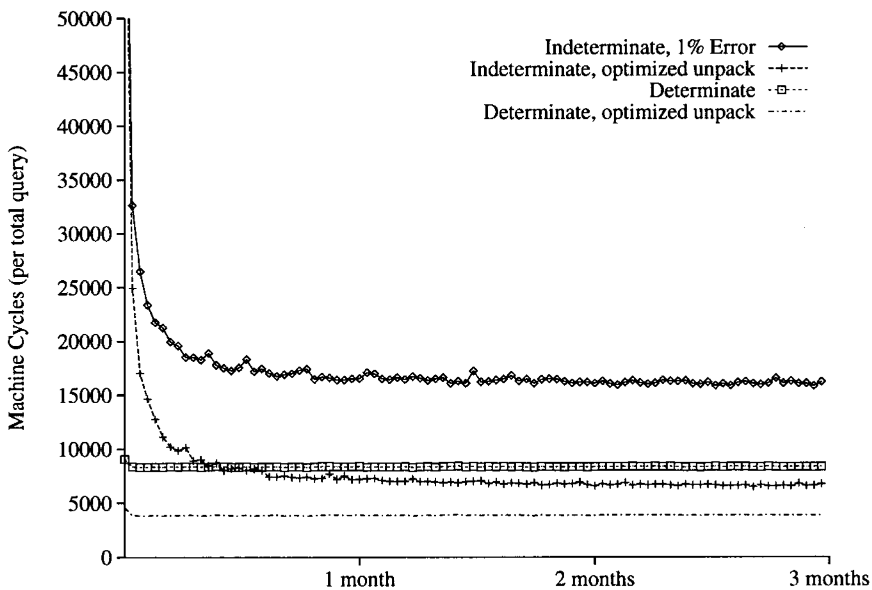


Fig. 17. The average cost of the example query per combination of tuples.

periods are unpacked. An alternative query processing strategy is to unpack each instant and period and cache the results, costing only $O(n)$ unpacks. We also programmed this sequence of calls, which we refer to as the "optimized unpack sequence." For all the sequences, we suppressed all input, output, and disk I/O, as these expensive operations tend to dominate the cost of other operations, and also because those costs are identical whether indeterminacy is present or absent.

To test the query, we once again used a variation of "the random placement of instants." We used the tuples shown in Figure 1, but randomly placed the instants in a chronon space of increasing size. We used a

	Warehouse	*Lot_Num*	*Part*	*When*
s_5	Boeing	40	*unknown*	May 31
s_6	Cessna	70	*some electrical part*	May 31
s_7	Boeing	41	{yoke, throttle}	May 31

Fig. 18. Examples of value incompleteness.

period of indeterminacy of one day (86,400 chronons) for the instants in both relations. The results are shown in Figure 17. Except for the rare situations where most of the instants are packed into a relatively tight random space, indeterminacy roughly doubles the CPU cost of the query.

10. EXTENDING TEMPORAL QUERY LANGUAGES

In SQL, valid-time indeterminacy is a form of *value incompleteness* (cf. [Motro 1990], where the value of an attribute is not fully known. The *Received* relation in Figure 18 provides examples of value incompleteness for the *Part* attribute: a part may exist which we have yet to identify (s_5), has been partially identified (s_6 restricts the kind of part to belong to the specified class of parts), or has been narrowed down to a set of possibilities (s_7). We showed in Section 3 how to support value incompleteness in instant, period, and interval attributes.

Valid-time indeterminacy can also apply in temporal data models [Tansel et al. 1993]. In such models, the values themselves vary over time. This behavior is often modeled by associating a timestamp with each tuple. Allowing this implicit timestamp to be indeterminate adds a new source of incompleteness, which we term *tuple valid-time indeterminacy*, which is orthogonal to other sources of incompleteness. In particular, it can peacefully coexist with value incompleteness, of both nontemporal and temporal attributes, and with *tuple incompleteness*, where the membership of a tuple in a relation is not fully determined. We advocate separating the various kinds of indeterminacy, so that users can choose the combination that is most appropriate for their application.

As a concrete example, we now review how tuple valid-time indeterminacy can be added to a specific temporal query language, TSQL2. This language was designed by a committee of eighteen researchers from academia, vendor research labs, and industry [Snodgrass 1995]. In this section, we consider a variant of TSQL2 that is being proposed for incorporation into the SQL3 standard [Snodgrass et al. 1996]. This variant differs from TSQL2's data model in allowing duplicate tuples and timestamping tuples with periods instead of with temporal elements (which are sets of maximal periods).

TSQL2 supports three temporal dimensions: valid time, transaction time, and user-defined time [Snodgrass 1995]. User-defined time is an uninterpreted time domain, having no special query language support. The approach to value incompleteness discussed to this point applies to user-

defined time. In the remainder of this section, we consider tuple valid-time indeterminacy. Specifically, we allow the period valid timestamp of a tuple to be an indeterminate period. As the timestamp indicates when the fact represented by the tuple was valid in reality, an indeterminate timestamp indicates that it is not known precisely when the fact became true, or no longer was true. Note, however, that we *do* know that the fact was true at some point (orthogonally, tuple incompleteness could be supported, to add the uncertainty in whether the fact was ever true).

10.1 Syntactic Extensions

Since TSQL2 is an extension of SQL-92, we start with the syntactic extensions introduced in Section 4, specifically, indeterminate temporal attributes in the create table and alter table statements, correlation credibility in the from clause, and ordering plausibility in the where clause. Interestingly, these are all that are necessary to add tuple valid-time indeterminacy to TSQL2. As an example, to define the *Received* relation as an indeterminate valid-time relation, we simply specify an indeterminate period type as the implicit timestamp.

```
CREATE TABLE Received (Warehouse CHARACTER(30),
                       Lot_Num   INTEGER,
                       Part      CHARACTER(40))
AS VALIDTIME INDETERMINATE PERIOD(DATE) WITH NONSTANDARD
DISTRIBUTION;
```

Note that the `When` attribute is no longer present; instead the timestamp is implicit. The implicit timestamp associated with a correlation name (e.g., `r`) is accessible within queries via the function `VALIDTIME(r)`; its value is impacted by the correlation credibility and ordering plausibility identically to the explicit values in the tuple.

10.2 Semantic Extensions

The semantics of TSQL2 is specified in terms of the semantics of SQL-92 [Snodgrass et al. 1996]. Since TSQL2 with tuple valid-time indeterminacy is an extension of SQL-92 with value valid-time indeterminacy, the major change is to replace $[\![]\!]_{SQL}$ in the TSQL2 semantics with $[\![]\!]_{ind}$.

TSQL2 has three modes, specified using new reserved words: *temporal upward compatibility*, in which only the current state is used, *sequenced semantics*, in which the query is applied with SQL semantics at each point in time, and *nonsequenced semantics*, in which the table is treated as a conventional table with an additional period attribute.

The nonsequenced semantics is already dealt with in the rest of the paper, with the proviso that the correlation credibility may replace the implicit period timestamp with a determinate value. In temporal upward compatibility, all tables are snapshot as of now. We use the overlap operator to determine those tuples valid at now; the operator is already defined for indeterminate periods. The specified plausibility, or the default if a plausibility is not specified, is used for the overlap. For the sequenced

semantics, a tuple will be considered to be valid at a given time point if the indeterminate period overlaps with the given time point, using the specified plausibility.

10.3 Semantics of Constructors

The final change required is support for *constructors*. A constructor is an operator that constructs a time value from one or more existing time values. Common constructors in temporal query languages include *First*, which constructs the earliest instant from a pair of instants; *Last*, which constructs the latest instant from a pair of instants; and *Period*, which constructs a period from a pair of instants. In this section, a general strategy to support constructors is developed. We first describe a specific strategy for the *Coalesce* constructor, and then generalize it to support any constructor.

10.3.1 *Coalescing*. The user can specify in ATSQL2 that a relation be *coalesced*: tuples with identical values for the explicit attributes (termed *value-equivalent*) with timestamps that overlap or meet in valid time are merged into a single tuple with a timestamp that is the union of the timestamps of the original tuples. If the periods do not overlap or meet, then the tuples are unchanged [Böhlen et al. 1996]. The semantic function *Coalesce* repeatedly coalesces overlapping pairs, until there is no more overlap.

In the indeterminate semantics, the coalescing operation must be extended to handle periods with indeterminacy.[9]

Indeterminate Coalescing. For two value-equivalent tuples that have valid-time periods $[\alpha,\beta]$ and $[\eta,\delta]$ that overlap with plausibility 100 (i.e., definitely overlap), the indeterminate coalescing of the periods $[\alpha,\beta]$ and $[\eta,\delta]$ produces the period $[s, t]$ where

$$s = (\mathbf{min}(\alpha_*, \eta_*) \sim \mathbf{min}(\alpha^*, \eta^*), P_{starting})$$

and

$$t = (\mathbf{max}(\beta_*, \delta_*) \sim \mathbf{max}(\beta^*, \delta^*), P_{terminating}),$$

such that

$$P_{starting}(x) = \mathbf{max}(F_\alpha(x), F_\eta(x)) - P_{starting}(x - 1)$$

where $\forall x < \mathbf{min}(\alpha_*, \eta_*)(P_{starting}(x) = 0)$, and

$$P_{terminating}(x) = P_{terminating}(x - 1) - \mathbf{max}(F'_\beta(x), F'_\delta(x)),$$

[9]This coalesce operator contends with indeterminate attribute values, and so differs from Dey and Sarkar's *coalescence* operation, which handles indeterminate *tuples* [Dey and Sarkar 1996].

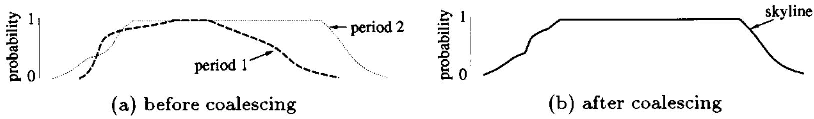


Fig. 19. Coalescing two indeterminate periods follows the skyline.

where $\forall x < \mathbf{min}(\beta_*, \delta_*)(P_{terminating}(x) = 1)$. F_i represents the cumulative density function associated with P_i, that is, $F_i(x) = \Sigma_{k \leq x} P_i(k)$ and F'_i is the complementary cumulative density function. If the periods do not definitely overlap, then the periods are already coalesced.

While this definition may appear complicated, the underlying idea is simple. Figure 19(a) shows a probability "profile" for two independent, overlapping periods. The profile for each is effectively just a plot of the cumulative and complementary cumulative density functions for the period's starting and terminating instants, respectively. Both periods 'ramp up' to definite information during their respective starting instants and 'fall off' to no information during the terminating instants. The result of coalescing the two periods is shown in Figure 19(b). The result is the "skyline" of the two periods. The skyline represents the maximal extent of both the determinate and indeterminate portions of the periods. This corresponds to the definition given above, which stipulates that the maximal cumulative or complementary cumulative density function value is used.

10.3.2 *Implementation of Coalescing*. The implementation techniques developed in Section 8 apply directly to the indeterminate portions of TSQL2, except for constructors such as coalescing, which require a new approach. A straightforward implementation of indeterminate coalescing could be very costly since, in some cases, the operator must dynamically compute a new probability mass function. Not only would dynamically computing such functions quickly exhaust the limited space of probability mass function names (recall from Section 7 that the timestamp formats limit the number of mass functions to 2^{16} possibilities), but the efficient implementation of $Before_I$ would also suffer since it depends upon the precomputed approximation of the mass function.

Our solution is to accept some information loss during coalescing and avoid the expense of dynamically computing a mass function by substituting the missing distribution for the new mass function. In other words, for two valid-time periods $[\alpha,\beta]$ and $[\eta,\delta]$ which definitely overlap, the implementation of indeterminate coalescing produces the period $[s, t]$ where

$$s = \begin{cases} \alpha & \text{if for every time } t,\ F_\alpha(t) \geq F_\eta(t) \\ \eta & \text{if for every time } t,\ F_\eta(t) \geq F_\alpha(t) \\ (\mathbf{min}(\alpha_*, \eta_*) \sim \mathbf{min}(\alpha^*, \eta^*), \mathbf{missing}) & \text{otherwise} \end{cases}$$

```
function First(in α, β : instant) : instant;
    begin
        if α* < β* then return α
        else if α* > β* then return β
        else if P_α is P_β then
            if α* < β* and α* < β* then return α
            else if α* > β* and α* > β* then kreturn β
            else return (min(α*, β*) ~ min(α*, β*), missing)
    end; { First }

function Coalesce(in [α, β], [η, δ] : period) : set-of-periods;
    var
        s, t : instant;
    begin
        if not overlap(100, [α, β], [η, δ]) then return {[α, β], [η, δ]}
        else return {[First(α, η), Last(β, δ)]}
    end; { Coalesce }
```

Fig. 20. Indeterminate Coalescing.

and

$$s = \begin{cases} \beta & \text{if for every time } t,\ F'_\beta(t) \geq F'_\delta(t) \\ \delta & \text{if for every time } t,\ F'_\delta(t) \geq F'_\beta(t) \\ (\mathbf{max}(\beta_*,\ \delta_*) \sim \mathbf{max}(\beta^*,\ \delta^*),\ \mathbf{missing}) & \text{otherwise} \end{cases}$$

But even this "weakened" version of coalescing is much too expensive, since the cumulative density functions for two instants must be computed for each chronon in a period of indeterminacy to determine if one function dominates another. However, it is easy to determine dominance in some special, but common, cases. For instance, for a pair of starting instants, if both instants have the same probability mass function, and one instant's period of indeterminacy starts before the other's ends, then the earlier instant is dominant. For example, the starting instant (1 $\sim$ 5, uniform) dominates the instant (3 $\sim$ 6, uniform). The implementation of indeterminate coalescing in *Coalesce'* only checks for dominance in these special cases, and substitutes the missing mass function in all other cases, as shown in Figure 20. Here only the *First* function is shown; this function chooses the earliest instant among a pair of instants. The *Last* function is analogous.

10.3.3 *A General Strategy for Supporting Constructors.* The general strategy for supporting constructors on indeterminate time values mimics the strategy used to implement *First*. The indeterminate *First* constructor differs from the determinate constructor only when the argument instants overlap; in which case, a new instant is constructed. While it is trivial to construct the period of indeterminacy for the new instant from the supports

for the underlying instant, in some cases a new mass function must also be constructed. But these functions are expensive to construct on the fly. Therefore, the missing mass function is used instead, although this substitution loses some information.

As an example, the INTERSECT operator can be formalized as follows:

$$[\![\texttt{PERIOD}(\langle inst_1\rangle, \langle inst_2\rangle)\texttt{INTERSECT PERIOD}(\langle inst_3\rangle, \langle inst_4\rangle)]\!]_{op} =$$

$$\texttt{PERIOD}(Last([\![\langle inst_1\rangle]\!]_{SQL}, [\![\langle inst_3\rangle]\!]_{SQL}),$$

$$First([\![\langle inst_2\rangle]\!]_{SQL}, [\![\langle inst_4\rangle]\!]_{SQL}))$$

More complicated constructors can be implemented in terms of *First* and *Last*.

So in general, a constructor will either result in an existing time value or produce a new time value. The behavior depends on whether the periods of indeterminacy overlap. If the periods of indeterminacy are disjoint, then the constructor simply chooses an existing time value. It is only when the periods overlap that a new time value must be constructed.

10.4 Summary

To support tuple valid-time indeterminacy in the variant of TSQL2 being proposed for SQL3, few syntactic extensions beyond those discussed for conventional SQL, presented in Section 4, are needed. The only significant semantic extension is to support constructors. We speculate that our approach to valid-time indeterminacy, both the value incompleteness variety and the tuple valid-time incompleteness variety, can be added in a similarly straightforward manner to any of the many temporal query languages proposed in the literature.

11. RELATED WORK

Despite the wealth of research on adding incomplete information to databases [Dyreson 1997; Parson 1996], there are few efforts that address incomplete temporal information. Much of the previous research in incomplete information databases has concentrated on issues related to null values [Vassiliou 1979; Zaniolo 1984; Codd 1990; Date 1986]. Another primary research thrust has studied the applicability of fuzzy set theory to relational databases [Dubois et al. 1988; Zemankova and Kandel 1985; Prade 1993]. There is also extensive AI literature on integrating various combinations of probabilistic reasoning, temporal reasoning, and planning; Kraus and Subrahmanian [1994] provide a nice summary of that literature.

We first place our work in the context of *value* and *tuple* incompleteness, then examine in detail several papers that concern *temporal* incompleteness.

Information that is valid-time indeterminate is similar to disjunctive information, especially in the context of deductive databases [Liu and

Sunderraman 1990]. Disjunctive information is a collection of facts, one (or more) of which is true. A set of possible chronons is of the exclusive-OR variety of disjunctive information (only one disjunct is true) [Ola 1992]. Valid-time indeterminacy differs from the above investigations because the alternatives are "weighted" and the weights are integrated into the query semantics.

The field of probabilistic databases covers a wide spectrum of different uses of probabilistic information. Probabilistic weights have been attached to attribute values to model situations where an attribute value could be one of several more or less likely values [Barbará et al. 1990; Barbará et al. 1992; Fuhr and Rölleke 1997; Gelenbe and Hebrail 1986; Lakshmanan et al. 1997; Lee 1992; Tseng et al. 1993]. Probabilistic weights have also been appended to tuples, where the weight is the probability that the tuple belongs to the relation [Cavallo and Pittarelli 1987; Dey and Sarkar 1996; Fuhr and Rölleke 1997; Kornatzky and Shimony 1993b; Kornatzky and Shimony 1993a; Lakshmanan et al. 1997; Lee 1992; Tseng et al. 1993; Zimányi 1992]. Decision support systems, vague queries, information retrieval and data mining have also utilized probabilistic information [Wong 1982; Fuhr 1990; Henrion and Suermondt 1993; Vasanthakumar et al. 1996]. Our work concerns only probabilities in attribute values and can be seen as an extension of the Probabilistic Data Model (PDM) [Barbará 1992]. In the PDM, attribute values are sets with weights attached to each element. The weight is the probability that the corresponding element is *the* value of the attribute. Queries use the probabilistic representation in conjunction with a single user-given "confidence" to compute a result within the framework of the possible world semantics. The novelty in our work can be seen in the methods used to retrieve the incomplete information and in how that information is represented. In the PDM, each element in a set of possible values is stored and processed separately. The costs of the probabilistic operators in PDM are proportional to the number of alternatives in the set (some of the operations have a cost that is proportional to the square of the number of alternatives).

Dey and Sarkar [1996] extend this data model, in part to render it in first normal form and to permit a more general join operation. More recently, Lakshmanan et al. [1997] have extended the PDM to eliminate several of the assumptions, including independence of elements, in their ProbView system. In particular, they permit a range of Cartesian product operators to be defined, with the situation dictating the appropriate strategy.

We could not adopt the PDM approach or its successors to support temporal indeterminacy, since there might be several million elements in a set of possible chronons. Representing each alternative with an associated probability is impractical. Due to the unique nature of valid-time indeterminacy, a different approach was required.

We now turn to the literature of temporal incompleteness, which, unlike our approach, does not employ probabilities.

In the earliest work on incomplete temporal information, an indeterminate instant was modeled with a set of possible chronons [Snodgrass 1982].

Before was extended to return the value *unknown*, necessitating an extension to a three-valued logic. Also, a four-valued logic was proposed to model times and values that are unknown, imprecise, or *negative* (under the open world assumption) [Schiel 1987]. Our current approach allows a probability mass function to be associated with each indeterminate instant, and does not require a multi-valued logic, though we do use such a logic in the operational semantics.

Dutta [1989] uses a fuzzy set approach to handle *generalized temporal events*. A generalized temporal event is a single event that has multiple occurrences. For example the event "Margaret's salary is high" may occur at various times as Margaret's salary fluctuates to reflect promotions and demotions. The meaning of "high" is incomplete because it is not a crisp predicate. In Dutta's model all the possibilities for "high" are represented in a *generalized* event and the user selects some subset according to his or her interpretation of "high." This contrasts with the task of encoding the type of information we have characterized as valid-time indeterminate. We view events as having a single occurrence. An indeterminate instant is a set of alternatives, one and only one of which is the actual time. Every member in a fuzzy set is always possible, to a greater or lesser extent, depending on the degree of membership, but always possible (although some fuzzy databases stipulate by fiat that only one member is possible [Dubois et al. 1988]. Our approach and that of Dutta's are representative of different kinds of temporal incompleteness. We feel that a probabilistic approach is better suited to modeling valid-time indeterminacy, as formulated in this paper, and that fuzzy set approaches like Dutta's [Vitek 1983; Dubois and Prade 1989] are better suited to modeling generalized events. The two approaches are orthogonal, and the user may pick the one(s) most appropriate to her application.

Generalized bitemporal elements are defined somewhat differently in more recent papers [Kouramajian and Elmasri 1992; Kouramajian et al. 1994]. Bitemporal elements combine transaction time and valid time in the same temporal element. Since TSQL2 also supports transaction time, valid-time indeterminacy and generalized bitemporal elements differ mainly in their handling of valid time. In Kouramajian and Elmasri's model, both the upper and lower support on a valid time period could be a set of noncontiguous possible chronons. Unlike valid-time indeterminacy, no probabilities are used. Since there are no probabilities, the user in general is limited to querying for answers that are either definite or those that are possible (or combinations thereof). Generalized valid times are composed of valid times by the operators of alternation (only one valid time applies) and/or union (both valid times could apply). We provide no capability for "generalizing" valid times to handle alternation or union.

Another proposed model intertwines support for value and temporal incompleteness [Gadia et al. 1992]. By combining the different kinds of incomplete information, a wide spectrum of attribute values are simultaneously modeled, including values that are completely known, values that are unknown but are known to have occurred, values that are known if

they occurred, and values that are unknown even if they occurred. In our approach, value incompleteness, tuple incompleteness, and tuple valid-time indeterminacy are orthogonal. By combining valid-time indeterminacy with other kinds of incomplete information, we can support each of the kinds of incomplete information found in Gadia et al., plus others (e.g., fuzzy value incompleteness). Another difference between our approach and theirs is that they make no use of probabilistic information. The user cannot express his or her credibility in the underlying data nor plausibility in the temporal relationships in the data.

Gadia et al. prove reliability for their model, as we did for ours, in Section 5.8. They also showed that, except for certain cases of selection, their operators were maximal. The same holds for our operators. Hence, both of our models (with the exception of certain cases of selection in their language) are theoretically sound.

While the possible and definite limits to incomplete temporal information are well understood, the *cost* of querying incomplete temporal information can be prohibitive. Satisfaction of general constraints, that is, a well-formed formulæ consisting of temporal variables, predicates and Boolean connectives (including negation), and allowing substitution of indeterminate instants is, not surprisingly, NP-hard [van Beek 1991; Dean and Boddy 1988].

Koubarakis [1993] showed that by restricting the kinds of constraints allowed, polynomial time algorithms can be obtained. Koubarakis proposed a temporal data model with global and local inequality constraints on the occurrence time of an event. The resulting model supports *indefinite* instants. An indefinite instant is a very general kind of instant that includes indeterminate instants, instants with disjoint sets of possible chronons, and instants with incompletely specified upper and lower supports. For instance, we may know that α occurred before β but after 2 PM (2 PM $< \alpha < \beta$), and we may know that β happened before 4 PM. In Koubarakis' data model, we can then conclude that α happened between 2 and 4 PM. Koubarakis restricts the kinds of constraints allowed (to disjunctions of inequalities) and achieves polynomial time complexity for retrieving information.

Another constraint-based temporal reasoner is LaTeR, which has been successfully implemented [Brusoni et al. 1995]. LaTeR similarly restricts the kinds of constraints allowed (to conjunctions of linear inequalities), but this class of constraints includes many of the important temporal predicates. LaTeR must construct a constraint network during insertion or updating of temporal information, but uses this network very efficiently when retrieving information to achieve $O(n^3)$ performance.

The primary difference between these constraint-based models and ours (other than probabilities) is that SQL (like most relational database models) does not allow inter-tuple constraints. Tuples in SQL relations are "row-independent", that is, no information is shared between tuples. Since the indeterminate data model is based on SQL, it makes no overt provisions

for sharing indeterminate information between tuples. Only the constraints given in the query are used to relate tuples. Further, we, like most others, assume that the variables in a query are independent. Our motivation is to avoid the expensive computation of dependent probabilities, but the independence assumption also yields an $O(nr)$ complexity for evaluating an unrestricted query consisting of temporal predicates and Boolean connectives (where n is the number of predicates in the query, and r is the number of tuples). However, constraint-based models are able to model temporal information that we cannot.

We note that there is little discussion in most of the aforementioned papers, save LaTeR, on implementation aspects. We feel that both efficient representations and efficient query-processing algorithms are essential, especially when the incomplete information is weighted.

12. SUMMARY AND FUTURE WORK

This paper has extended the syntax and formal semantics of SQL to support valid-time indeterminacy and has described an efficient implementation for that support. We also showed how the concepts can be applied to temporal query languages, in particular to the variant of TSQL2 being proposed for incorporation into SQL3.

We return to the goals enumerated in Section 1. The syntactic extensions are minimal. In particular, we provide the user with two controls on the retrieval process: correlation credibility and ordering plausibility. We have augmented the create table and alter table statements to specify which attributes incorporate valid-time indeterminacy and to identify which timestamp format to use, extended the from clause to specify correlation credibility using an optional with clause, extended the select statement to specify ordering plausibilities, and added variants to the set statement to specify default plausibilities and credibilities. Correlation credibility changes the information available to query processing; it replaces indeterminate with determinate information. Ordering plausibility controls the construction of an answer to a query using the pool of credible information. A temporal expression is satisfied if there exists a plausible ordering (to the level specified by the user) that satisfies it. The approach is orthogonal to those proposed by others to handle value incompleteness and generalized events (cf. Section 10), has an intuitive semantics (Section 5), retains the first normal form structure of SQL and TSQL2 (Section 3), refines previously proposed techniques to handle multiple granularities of time (Section 7), is temporally upward compatible, in that it reduces to SQL's and TSQL2's semantics in the absence of indeterminacy (Section 5.3), reduces to SQL's semantics even in the presence of indeterminacy, when the new constructs are not used (Section 5.6), and has been proven to be both reliable and maximal (Section 5.8).

While these language extensions are highly expressive, this paper demonstrates that they can be efficiently implemented. We showed how indeterminate instants with a uniform probability mass function or mass

function that is missing can be represented in only 64 bits in most cases; for user-defined distributions the common representation is only 96 bits (Section 7). The operational semantics was shown to be correct (Section 5.8); the rod-counting algorithm is efficient (Section 8.1.4), and the under-counting of this algorithm is minimal (also Section 8.1.4). We implemented the functions required by the altered semantics (Section 8), and demonstrated that the implementation roughly doubles the CPU cost of a query when indeterminacy is present and plausibilities and credibilities less than 100 are specified, while having little impact on its disk I/O time (Section 9).

This paper only considers the select, create table, and alter table statements of SQL and TSQL2. The modification statements (append, delete, and update) as well as views, integrity constraints, assertions, and cursors, can be extended in an analogous manner. The approach espoused here has been adopted in TSQL2 [Dyreson and Snodgrass 1995b].

One important area of future research is the extension of indexing methods to support indeterminate values. Most extant indexing approaches assume a total order on the underlying domain. Only for an ordering plausibility of 100 does such a total order exist for indeterminate instants (cf. Section 5.4.1). Indexing methods need to be extended to support arbitrary ordering plausibilities, to be specified at query time.

Algorithm improvements on the functions discussed in Section 8 are certainly possible. While such improvements might reduce execution time considerably in special cases, we doubt that they will have much impact on the average case.

A useful extension of the current work would be to use periods instead of values to express plausibility. For instance, the user could constrain retrieval to tuples that "overlap March 1984" to "within a year" (this has been termed a "band join" [DeWitt et al. 1991] or a "fuzzy temporal equi-join" [Leung and Muntz 1991]). This possibility can be seen as an extension of the present paper, specifically, as a refinement of the $Before_I$ operation.

Another useful extension would be to annotate the returned periods with a "plausibility ranking," thereby obviating the need for a plausibility clause. Computing such a ranking would complicate query evaluation.

Finally, it is always useful to consider increasing the expressive power of the data model and query language. In particular, one important assumption we make throughout is that tuples are row-independent, with no information shared between indeterminate tuples. Most of the other approaches that utilize probabilities to model various flavors of incompleteness make this assumption as well, because computing dependent probabilities in the inner loop of query processing is just too expensive. We also assume that indeterminate instants are modeled by contiguous sets of possible chronons. We do not support noncontiguous sets that could model indeterminate instants such as "it happened yesterday morning or this morning." We exploit both of these assumptions to achieve efficiency in representation and in query processing. We conjecture that relaxing either of these assumptions will cause the query evaluation complexity to return

to cubic (or worse), thereby rendering the performance unacceptable for many temporal database applications.

APPENDIX: Proofs of Theorems

THEOREM 1. $[\![\mathit{WHERE}\ \langle\text{predicate}\rangle]\!]_{ind}(1, r)$ *is reliable.*

PROOF. For any where clause W, for all $r' \in \mathbf{C}(r)$, let $c = [\![W]\!]_{SQL}(r'^{[prime]})$. Then $\forall t' \in r'(t' \in c \Rightarrow [\![\langle predicate\rangle]\!]_{SQL}(t'))$. From the definition of $[\![W]\!]_{ind}$,

$$\forall t' \in r'(t' \in c \Rightarrow (\exists t \in r(t' \in \mathbf{C}(t) \wedge t \in [\![W]\!]_{ind}(1, r))))$$

From the definition of $\mathbf{C}$, it follows that $c \in \mathbf{C}([\![W]\!]_{ind}(1, r))$. □

THEOREM 2. $[\![\mathit{WHERE}\ \langle\text{predicate}\rangle]\!]_{ind}(1, r)$ *is maximal.*

PROOF. We need to show $\forall\langle where\ clause\rangle W, \forall c \in \mathbf{C}([\![W]\!]_{ind}(1, r)), \exists r' \in \mathbf{C}(r)(c = [\![W]\!]_{SQL}(r'))$. For the possible interpretation,

$$\forall\langle where\ clause\rangle W, \forall c \in \mathbf{C}([\![W]\!]_{ind}(1, r))$$

$$(\forall t' \in c, \exists t \in r(\exists t'' \in \mathbf{C}(t) \wedge [\![P]\!]_{SQL}(t''))).$$

From the definition of $\mathbf{C}$, it follows that $\exists r' \in \mathbf{C}(r)(c = [\![W]\!]_{SQL}(r'))$. □

THEOREM 3. $[\![]\!]_{ind}$ *and* $[\![]\!]_{op}$ *agree on the possible and definite interpretations.*

PROOF. For the definite bound ($\gamma = 100$), we need to show

$$\forall\langle predicate\rangle P, \forall r, \forall d([\![\text{WHERE } P]\!]_{ind}(100, r)$$

$$= \{t \mid t \in r \wedge T \in [\![P]\!]_{op}(100, r)\}).$$

From the definition of $[\![]\!]_{ind}$, this is equivalent to $\forall t \forall c \in \mathbf{C}(t)([\![P]\!]_{SQL}(c) \Rightarrow T \in [\![P]\!]_{op}(100, t))$. We show this by induction on the Boolean connectives in P, after replacing P with its equivalent in terms of *Before* (or $Before_I$ for $[\![]\!]_{op}$), $\wedge$ and $\neg$.

Basis: $[\![P]\!]_{SQL}$ is *Before(a,b)* where a and b are (instant) values in the tuple t. From the definition of the completion of a tuple, we must show

$$(\forall c_a \in \mathbf{C}(a), \forall c_b \in \mathbf{C}(b)(c_a \leq c_b)) \Rightarrow T \in Before_I(a, b, 100).$$

The left-hand side implies that $\mathbf{Pr}[a \leq b] = 1.00$, so $Before_I(a, b, 100) = \{T\}$.

Induction: Assume that $\forall t \forall c \in \mathbf{C}(t)(\llbracket P \rrbracket_{SQL}(c) \Rightarrow T \in \llbracket P \rrbracket_{op}(100, t))$ holds for P containing $i - 1$ Boolean connectives. We need to show that this holds for P containing i connectives.

(1) P is $P_1 \wedge P_2$. Applying the definitions of $\llbracket\rrbracket_{SQL}$ and $\llbracket\rrbracket_{op}$, the following holds from the inductive hypothesis.

$$\forall t \forall c \in \mathbf{C}(t)(\llbracket P_1 \rrbracket_{SQL} \wedge \llbracket P_2 \rrbracket_{SQL}(c)$$
$$\Rightarrow T \in (\llbracket P_1 \rrbracket_{op}(100, t) \cap \llbracket P_2 \rrbracket_{op}(100, t)))$$

It then follows that the inductive hypothesis also holds for P.

(2) P is $\neg P_1$. Again, applying the definitions to the inductive hypothesis,

$$\forall t \forall c \in \mathbf{C}(t)(\neg \llbracket P_1 \rrbracket_{SQL}) \Rightarrow F \in (\llbracket P_1 \rrbracket_{op}(100, t)).$$

This shows that the two semantics agree for P containing i predicate connectives.

For the possible bound ($\gamma = 1$), we need to show

$$\forall \langle predicate \rangle\, P, \forall r, \forall d(\llbracket \texttt{WHERE } P \rrbracket_{ind}(1, r) = \{t \mid t \in r \wedge T \in \llbracket P \rrbracket_{op}(1, r)\}).$$

This is equivalent to $\forall t \exists c \in \mathbf{C}(t)(\llbracket P \rrbracket_{SQL}(c) \Rightarrow T \in \llbracket P \rrbracket_{op}(1, t))$. Comparing this to the analogous formula in the definite portion of this proof, note that $\forall$c, and $\gamma = 100$ have been replaced with $\gamma = 1$.

Again, we show this by induction on the number of Boolean connectives in P.

Basis: $\llbracket P \rrbracket_{SQL}$ is *Before*(a, b) where a and b are (instant) values in the tuple t. From the definition of the completion of a tuple, we must show

$$(\exists c_a \in \mathbf{C}(a), \forall c_b \in \mathbf{C}(b)(c_a \leq c_b)) \Rightarrow T \in Before(a, b, 1).$$

Applying the definition of $Before_I$, the right-hand side is equivalent to $\mathbf{Pr}[a \leq b] > 0$ (here we use $\mathbf{Pr}'[\ \ldots\ ] = 0.01$). From the left-hand side, we know that $\mathbf{Pr}[a = c_a] > 0$ and $\mathbf{Pr}[b = c_b] > 0$, so $\mathbf{Pr}[a \geq b] > 0$. Hence,

$$(\exists c_a \in \mathbf{C}(a), \exists c_b \in \mathbf{C}(b)(c_a \leq c_b))$$
$$\Rightarrow \mathbf{Pr}[a = c_a] > 0 \wedge \mathbf{Pr}[b = c_b] > 0 \wedge c_a \leq c_b$$
$$\Rightarrow \mathbf{Pr}[a \geq b] > 0$$
$$\Rightarrow T \in Before_I(a, b, 1)$$
$$\Rightarrow T \in \llbracket P \rrbracket_{op}(1, t).$$

Induction: We utilize a similar induction hypothesis as before, and the inductive steps follow naturally. □

THEOREM 4. *$[\![S]\!]_{ind}(\delta, \gamma, r)$ is monotonic in γ.*

PROOF. As the semantics of each clause depends only on one parameter, the proof proceeds in two parts: (1) show that $[\![\langle where\ clause\rangle]\!]_{ind}$ is monotonic in γ, and (2) show that $[\![\langle target\ list\rangle]\!]_{ind}$ is monotonic in γ.

$$\forall t \forall \langle predicate\rangle P(\gamma \geq \gamma' \Rightarrow \mathbf{C}([\![P]\!]_{op}(\gamma, t)) \leq \mathbf{C}([\![P]\!]_{op}(\gamma', t))).$$

We prove this by induction on the Boolean connectives in p, after replacing p with its equivalent, as discussed in Section 5.4.2.

Basis: $[\![P]\!]_{op}$ is $Before_I(a, b, \gamma)$. This follows from the definition of $Before_I$ and the monotonicity of **Pr**.

Induction: Assume that the above holds for P containing $i - 1$ Boolean connectives.

(1) P is P_1 AND P_2, which is replaced with $P_1 \cap P_2$, which is monotonic when P_1 and P_2 are monotonic.

(2) P is NOT P_1, which is replaced with $\{x \mid (\neg x) \in [\![P]\!]_{op}(\gamma, r)\}$. Again, this is monotonic if P_1 is monotonic.

To show (2), we observe that $[\![\langle target\ list\rangle]\!]_{ind}$ utilizes constructors based on $Before_I$. By Theorem 3, $[\![]\!]_{ind}$ is an equivalent semantics, and hence from (1), just shown, (2) follows. □

THEOREM 5. *The kth pivot will count $P^2/2^{\lfloor \log_2(k)\rfloor+1}$ pairs.*

PROOF. By choosing the rod corresponding to half of the remaining rods, the algorithm counts half the pairs on the first pivot, that is, it counts $P^2/2$ pairs. On the second and third pivots, it counts half of half of the remaining pairs, or $P^2/8$ pairs per pivot, assuming "breadth-first" recursion. On the fourth through seventh pivots, it counts half of half of half of the remaining pairs, or $P^2/32$ pairs. So, in general, the kth pivot will count $P^2/2^{2\cdot\lfloor \log_2(k)\rfloor+1}$ pairs. In the worst case, $2P$ pivots are required. □

THEOREM 6. *The undercount is less than $2/P$.*

PROOF. First consider the error once a pivot has been chosen. The error is the rods in the other row of rods that remain uncounted. The uncounted rods are those that overlap the pivot. These rods are uncounted because it is unknown how the probability mass is distributed within each rod; consequently, it is impossible to determine whether the mass is before or after the mass in the pivot. Figure 21 shows the rods that are uncounted for an example pivot; the rods that are either partially or wholly within the dotted lines are not counted. But how many pairs of rods possibly overlap? We claim that there can be at most $2P - 1$ pairs that overlap.

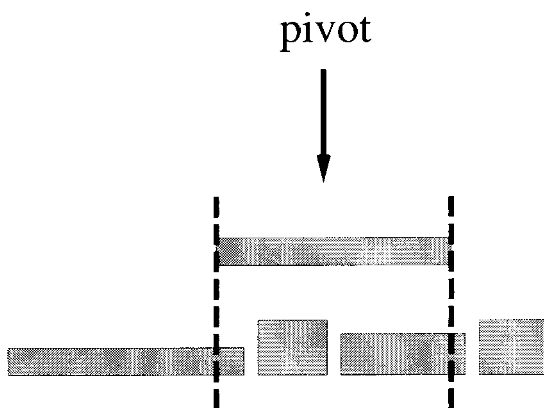


Fig. 21. The rods within the dotted lines are the undercount for the pivot.

We demonstrate this claim by modeling the overlapping rods with an undirected graph. Let each rod be a node in a graph. Add an edge between each pair of rods that overlaps. Observe that the edges cannot "cross" each other, that is, the graph is *planar*. Now count the total number of edges in the graph. Choose the first, or "leftmost" edge in the graph. Since edges cannot cross, at least one node on this edge is a sink, unconnected to any other nodes by a different edge. Eliminate both the node and the edge. Repeat this process, always choosing the "leftmost" remaining edge, until there are no more edges. Initially there are $2P$ nodes. One node is eliminated at every step along with one edge. At least one node remains after the final edge is removed. Consequently, at first, there were at most $2P - 1$ edges.

Each edge represents a pair of rods that overlap, corresponding to a mass of $1/P^2$ that remains uncounted. Since there are at most $2P - 1$ edges, the total missing mass is less than $2/P$. For a precision of 2^8, the total error is less than 1%. □

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