Valid-Time Indeterminacy in Temporal Relational Databases: Semantics and Representations

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Abstract—Valid-time indeterminacy is “don’t know when” indeterminacy, coping with cases in which one does not exactly know when a fact holds in the modeled reality. In this paper, we first propose a reference representation (data model and algebra) in which all possible temporal scenarios induced by valid-time indeterminacy can be extensionally modeled. We then specify a family of sixteen more compact representational data models. We demonstrate their correctness with respect to the reference representation and analyze several properties, including their data expressiveness. Then, we compare these compact models along several relevant dimensions. Finally, we also extend the reference representation and a representative of compact representations to cope with probabilities.

Index Terms—H.2.4.m Temporal databases, I.2 Artificial Intelligence, H.2.0.b Database design, modeling and management, I.2.4 Knowledge Representation Formalisms and Methods.

1 Introduction

Time is pervasive and in many situations the dynamics over time is one of the most relevant aspects to be captured by a data model. Many representations for temporal databases (TDBs) have been developed over the last two decades.

Valid-time indeterminacy (“don’t know when” information [9]) comes into play whenever the valid time associated with some piece of information in the database is not known in an exact way. Consider the following example (at a granularity of hours).

Example 1. On Jan 1 2010 between 1am (inclusive) and 4am (exclusive) John had breathing problems.

The fact “John had breathing problems” holds at an unknown number of time units (hours), ranging from hours 1 to 3 inclusive, i.e., it may hold on 1, 2, and 3, or on 1 and 3, or on 2 only, and so on. (For the sake of brevity, in this paper we denote by *n* the hour from *n* to *n+1*, and we assume to start the numbering of hours on Jan 1 2010).

As a border case, the fact that a given event might have occurred or not (i.e., indeterminacy about the *existence* of the fact) may be interpreted as a form of valid-time indeterminacy; consider:

Example 2. On Jan 1 2010 between 1am (inclusive) and 4am (exclusive) Mary might have had an ischemic stroke.

Coping with valid-time indeterminacy is important in many database applications, since the time when facts happen is often partially unknown. However, the treatment of valid-time indeterminacy has not received much attention in the TDB literature.

A commonly agreed-upon strategy to cope with time in relational databases is to extend the data model to associate temporal elements (i.e., sets of time points, or, equivalently, sets of time intervals) with tuples, and to extend relational operators to cope with such an additional temporal component. Specifically, temporal relational operators usually perform “standard” operations on the non-temporal component, and apply set operators on temporal elements (e.g., Cartesian product involves the intersection of the temporal elements of the tuples being paired). However, to the best of our knowledge, such a methodology has not yet been fully explored in the context of temporal indeterminacy (see the “Temporal Indeterminacy” entry in Liu and Tamer Özsu [19]). For example, the work by Dyreson and Snodgrass [9] only copes with periods of indeterminacy and does not provide set operators on them, nor temporal relational operators working on the extended representation. Additionally, to the best of our knowledge, no current approach copes with indeterminacy about existence.

We attempt here to overcome such limitations. Indeed, our goal is quite ambitious: we do not just aim to provide a specific representation for indeterminate temporal elements as well as set operators on them (plus the related temporal relational algebra), but to explore a wide range of representational possibilities. Indeed, in this paper we propose 17 different approaches to temporal indeterminacy. We extensively study the main properties of such approaches: (i) expressiveness, (ii) closure and correctness of algebraic operators, and (iii) whether the approaches are a consistent extension of BCDM [14] [20], a semantics adopted by many temporal database approaches. Finally, we compare such approaches, considering their expressiveness, their capability to cope with existential indeter-

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minacy, their *suitability* [15], intended as the "intuitive notion of expressiveness which takes the modelling effort into account" [22], and their computational cost.

1.1 Methodology

In this paper, we ground our approach on BCDM [14] [20]. We utilize a commonly-used methodology: (1) we first propose a reference approach coping with the phenomenon; and only then (2) we devise more user-friendly, compact, and efficient representations.

Our reference approach (data model and algebra) allows one to extensionally model (bringing to mind data expressiveness) and query (query expressiveness) all possible temporal scenarios induced by valid-time indeterminacy. We provide a consistent extension of BCDM, in the sense that determinate valid time can be easily coped with as a special case (thus granting for the compatibility and interoperability with existent approaches). However, (data/query) expressiveness is not the only criterion. It is also important to provide users with formalisms that model phenomena in a "suitable" and "compact" way.

We first identify four refinements (for example, one of them emphasizes suitability and compactness in coping with constraints about valid-time minimal duration). Each refinement is independently satisfied (or not). On the basis of these refinements, we propose a family of sixteen representations, each supporting a specific combination of such refinements in a more compact and user-friendly way (with respect to the reference approach). Each representation is characterized (i) by a different formalism to represent valid time, (ii) by the definition of set operations (i.e., union, intersection and difference) on the given representation of valid time, and (iii) by the relational algebra operations based on such set operations.

For each data representation, we study its semantics and (data) expressiveness with respect to the reference approach. We have defined the set operators within the different representations in such a way that they are proven to be correct with respect to the reference approach. Roughly speaking, this means that, although such operators operate on a more compact representation, they provide the same results as the reference approach. However, we proved that not all the sixteen representations could support a closed definition of set operators: in some representations, the correct result of set operations cannot be expressed in the representation formalism. Of course, only representations which support a closed definition of set operators —a *closed* representation for short— are suitable for DB applications.

For each "closed" representation, we define the relational algebraic operators as a polymorphic adaptation of the operators of the reference approach and determine whether each is a consistent extension of the BCDM operators. Finally, we also extend our approach to cope with probabilities.

This paper thus provides a family of representations of temporal indeterminacy overcoming the limitations of current approaches, as well as a formal framework which can be used in order to analyze and classify extant and potential representations for valid-time indeterminacy. Users can choose between such representations the best-suited approach to model their application domain.

The paper is organized as follows. In Section 2, we present our reference approach. In Section 3, we identify the four refinements for a compact representation, and we describe five representations: one for each refinement plus the representation resulting from the combination of all the refinements. Section 4 summarizes the results concerning also the other representations in the family. In Section 5, we extend both the reference approach and one of the compact representations to deal with probabilities. Finally, in Section 6 we propose comparisons and in Section 7 we draw some conclusions.

2 Reference Approach

In this section, we introduce the reference approach we propose to cope with temporal indeterminacy. Our starting point is BCDM [14].

2.1 BCDM

BCDM (Bitemporal Conceptual Data Model) [14] is a unifying data model, isolating the "core" semantics underlying many temporal relational approaches, including TSQL2 [14] [20]. In BCDM, tuples are associated with *valid time* and *transaction time*. For both domains, a limited precision is assumed (the *chronon* is the basic time unit). Both time domains are totally ordered and isomorphic to the subsets of the domain of natural numbers. The domain of valid times D_{VT} is given as a set $D_{VT}=\{c_1,\ldots,c_k\}$ of chronons, and the domain of transaction times D_{TT} is given as $D_{TT}=\{c'_1,\ldots,c'_j\}\cup\{UC\}$ (where *UC* –Until Changed– is a distinguished value). In general, the schema of a BCDM relation $R=(A_1,\ldots,A_n\,|\,T)$ consists of an arbitrary number of non-timestamp (*explicit* henceforth) attributes $A_1, \ldots, A_n$, encoding some fact, and of a timestamp attribute T, with domain $D_{TT}\times D_{VT}$; the explicit attributes and the timestamp attribute are separated by the symbol |. Thus, a tuple $x=(v_1,\ldots,v_n\,|\,t_b)$ in a BCDM relation $r(R)$ on the schema R consists of a number of attribute values associated with a set of bitemporal chronons $c_{bl}=(c'_h, c_i)$, with $c'_h\in D_{TT}$ and $c_i\in D_{VT}$, to denote that the fact $v_1,\ldots,v_n$ is current (present in the database) at time c'_h and valid at time c_i. An empty timestamp and value-equivalent [20] tuples are not admitted. Valid-time, transaction-time and atemporal tuples are special cases, in which either the transaction time, or the valid time, or both of them are absent. In the following, we restrict our attention to valid time (in fact, temporal indeterminacy cannot affect transaction time), and extend this general model to deal with temporal indeterminacy.

2.2 Disjunctive temporal elements

As in BCDM [14] (and in many approaches reviewed in [20]), in our approach time is totally ordered and isomorphic to the natural numbers. For the sake of simplicity, a single granularity (e.g., hour) is assumed.

Definition 1 Chronon. The chronon is the basic time unit. The chronon domain *TC*, also called timeline, is the ordered set of chronons $\{c_1, \ldots, c_i, \ldots, c_j, \ldots\}$ with $c_i<c_j$ as $i<j$.

As in BCDM, sets of chronons are used in order to associate with each tuple its valid time.

Definition 2 Temporal element. A temporal element is a set of chronons, i.e., an element of $PS(TC)$, the power set of TC.

Disjunctions of temporal elements are a natural way of coping with valid-time indeterminacy, in which each temporal element models one of the alternative possible temporal scenarios (any one of which could be valid).

Definition 3 Disjunctive temporal element, termed DTE. A disjunctive temporal element is a disjunctive set of temporal elements. Given a temporal domain TC, a DTE is an element of $PS(PS(TC))$.

For example, the following DTE models the valid time in Example 1: {{1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}}.

Notice that indeterminacy about existence can be simply modeled by including the empty temporal element within a DTE. Determinate times can be modeled through a DTE containing just one temporal element (called *singleton* DTE).

Property 1 Consistent extension (DTE). Any determinate temporal element can be modeled by a singleton DTE.

2.2 Temporal tuples and relations

To represent facts that are temporally indeterminate, DTEs are used as timestamps of the facts. Intuitively, DTEs cope with valid-time indeterminacy by explicitly modeling all the alternative temporal scenarios.

Definition 4 (valid-time) indeterminate tuple and relation. Given a schema $(A_1, \ldots, A_n)$ (where each A_i represents a non-temporal attribute on the domain D_i), a (valid-time) indeterminate relation r is an instance of the schema $(A_1, \ldots, A_n \mid VT)$ defined over the domain $D_1 \times \ldots \times D_n \times PS(PS(TC))$ in which empty valid times and value-equivalent tuples are not admitted (as in BCDM). Each tuple $x = (v_1, \ldots, v_n \mid d) \in r$, where d is a DTE, is termed a (valid-time) indeterminate tuple. The DTE $d = \{\{c_i,\ldots,c_j\}, \ldots, \{c_h,\ldots,c_k\}\}$ within tuple x denotes that the tuple x holds either at each chronon in $\{c_i, \ldots, c_j\}$ or … or at each chronon in $\{c_h, \ldots, c_k\}$.

Example 3. On Jan 1 2010 Sue might have had an ischemic stroke either at 1am or at 2am.

Example 4. On Jan 1 2010 Tim had breathing problems certainly at 1am and possibly at 2am or 3am.

CLINICAL_RECORD is a temporally indeterminate relation representing Examples 1–4.

CLINICAL_RECORD
{ (John, breath | {{1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}}),
(Mary, stroke | {∅, {1}, {2}, {3}}),
(Sue, stroke | {∅, {1}, {2}}),
(Tim, breath |{{1}, {1,2}, {1,3}, {1,2,3}}) }

The first tuple models Example 1. The second tuple models Example 2 considering the additional knowledge that the ischemic stroke, if any, has been unique and has occurred in –at most– one hour. ∅ represents that the fact might have not occurred. Finally, the third and fourth tuples model Examples 3 and 4 respectively.

2.3 Lattice of scenarios

The elements of $PS(TC)$ with the standard set inclusion relation form a lattice which represents the space of all possible alternative scenarios over the temporal domain TC. We term this a *lattice of scenarios* (over TC).

Property 2 Expressiveness. By definition, the formalism in this section allows one to express (i.e., to associate with each tuple) any combination of possible scenarios (i.e., any subset of the lattice of scenarios).

In Figure 1 we represent the lattice of scenarios considering the chronons {1,2,3} and the subsets of the lattice of scenarios represented by Examples 1, 2 and 3.

In Sections 3 and 4 we describe also less expressive (but more compact) formalisms, which in some cases cannot represent all possible combinations of scenarios (i.e., not all subsets of the lattice of scenarios).

2.4 Algebraic operations

Codd designated as complete any query language that was as expressive as his set of five relational algebraic operators: relational union (∪), relational difference (–), selection (σ_P), projection (π_X), and Cartesian product (×) [6]. Here we generalize these operators to cover (valid-time) indeterminate relations. As in several TDB models, our temporal operators behave as standard non-temporal operators on the non-temporal attributes, and apply set operators on the temporal component of tuples (see, e.g., Snodgrass [20]). As in many TDB models, including TSQL2 and BCDM, in our proposal Cartesian product involves the intersection of the temporal components, projection and union involve their union, and difference the difference of temporal components. (This definition can be motivated by a *sequenced* semantics [8]: results should be valid independently at each point of time.)

Now we define the relational operators of union ($\cup^{TI}$), difference ($-^{TI}$), projection ($\pi_X{}^{TI}$), selection ($\sigma_X{}^{TI}$) and Cartesian product ($\times^{TI}$) between temporally indeterminate relations. But, before doing so, we define the (generalized) set operators of intersection ($\cap^{DTE}$), union ($\cup^{DTE}$) and difference ($-^{DTE}$) applied to DTEs.

Definition 5 $\cup^{DTE}$, $\cap^{DTE}$, and $-^{DTE}$. Given two DTEs DA and DB, and denoting their temporal elements by A and B respectively $\cup^{DTE}$, $\cap^{DTE}$, $-^{DTE}$ between DA and DB are defined as the DTE obtained through the pairwise application of standard set operations on temporal elements:

$DA \cup^{DTE} DB = \{A \cup B \mid A \in DA \wedge B \in DB\}$

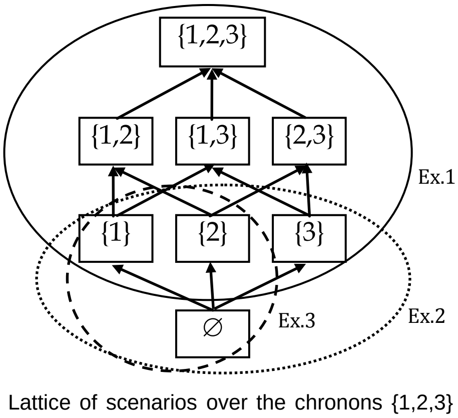

Figure 1. Lattice of scenarios over the chronons {1,2,3} ordered with respect to set inclusion. The solid-line oval, the dotted-line oval and the dashed-line oval represent the scenarios of Example 1, of Example 2 and of Example 3, respectively.

$DA \cap^{\mathrm{DTE}} DB = \{A \cap B \mid A \in DA \wedge B \in DB\}$
$DA -^{\mathrm{DTE}} DB = \{A - B \mid A \in DA \wedge B \in DB\}$.

Intuitively, DTEs represent valid-time indeterminacy by eliciting all possible alternative determinate scenarios. The rationale behind our definition is simply that the pairwise combination of each alternative scenario must be taken into account. For instance, considering the CLINICAL_RECORD relation, $\{\varnothing, \{1\}, \{2\}, \{3\}\} \cap^{\mathrm{DTE}} \{\varnothing, \{1\}, \{2\}\}$ identifies all times when both Mary and Sue had a stroke, and the final result is the set of scenarios obtained by combining each scenario for Mary and Sue through pairwise standard set intersection, i.e., $\{\varnothing\cap\varnothing, \varnothing\cap\{1\}, \varnothing\cap\{2\}, \{1\}\cap\varnothing, \{1\}\cap\{1\}, \{1\}\cap\{2\}, \{2\}\cap\varnothing, \{2\}\cap\{1\}, \{2\}\cap\{2\}, \{3\}\cap\varnothing, \{3\}\cap\{1\}, \{3\}\cap\{2\}\}$, which yields $\{\varnothing, \{1\}, \{2\}\}$. Hence, it is the case that (*a*) there was no time when both Mary and Sue had a stroke, or (*b*) they both had a stroke in hour 1, or (*c*) they both had a stroke during hour 2.

Definition 6 Temporal relational algebraic operators. Let *r* and *s* denote two (temporal) indeterminate relations on the proper schema. The temporal algebraic operators of union, difference, projection, selection and Cartesian product of *r* and *s* are defined as follows.

$r \cup^{TI} s = \{ <v \mid t> \mid$
$\quad \exists t_r\,(<v \mid t_r> \in r \wedge \neg\exists t_s\,(<v \mid t_s> \in s) \wedge t = t_r)$
$\quad \vee \exists t_s\,(<v \mid t_s> \in s \wedge \neg\exists t_r\,(<v \mid t_r> \in r) \wedge t = t_s)$
$\quad \vee (\exists t_r\,(<v \mid t_r> \in r) \wedge \exists t_s\,(<v \mid t_s> \in s) \wedge t = t_r \cup^{\mathrm{DTE}} t_s) \}$

$r -^{TI} s = \{ <v \mid t> \mid$
$\quad \exists t_r\,(<v \mid t_r> \in r \wedge \neg\exists t_s\,(<v \mid t_s> \in s) \wedge t = t_r)$
$\quad \vee \exists t_r\, \exists t_s\,(<v \mid t_r> \in r \wedge <v \mid t_s> \in s \wedge t = t_r -^{\mathrm{DTE}} t_s \wedge$
$\quad\quad t \neq \{\varnothing\}) \}$

$\pi_X{}^{TI}(r) = \{ <v \mid t> \mid$
$\quad \exists v_r\ t_r\ (< v_r \mid t_r > \in r \wedge v = \pi_X(v_r)) \wedge$
$\quad\quad t = \bigcup^{\mathrm{DTE}}{}_{<v_r \mid t_r> \in r \wedge v = \pi_X(v_r)}\, t_r \}$

$\sigma_P{}^{TI}(r) = \{ <v \mid t> \mid <v \mid t> \in r \wedge P(v) \}$

$r \times^{TI} s = \{ <v_r \cdot v_s \mid t> \mid$
$\quad \exists t_r\, \exists t_s\,(<v_r \mid t_r> \in r \wedge <v_s \mid t_s> \in s \wedge t = t_r \cap^{\mathrm{DTE}} t_s \wedge t \neq \{\varnothing\}$
$\quad\quad) \}$.

In addition to Codd operators, temporal selection can be added, to select tuples whose valid time *t* satisfies a selection condition φ. Interestingly, in the case of indeterminate temporal information, one may want to specify whether the condition φ(*t*) must necessarily (NEC) or possibly (POSS) hold (three-valued approaches have been widely used to cope with incomplete information in databases; consider, e.g., Gadia et al. [11]).

$\sigma_{\mathrm{NEC}\,\varphi}{}^{\mathrm{TI}}(r) = \{ <v \mid t> \mid <v \mid t> \in r \wedge \mathrm{NEC}(\varphi(t)) \}$
$\sigma_{\mathrm{POSS}\,\varphi}{}^{\mathrm{TI}}(r) = \{ <v \mid t> \mid <v \mid t> \in r \wedge \mathrm{POSS}(\varphi(t)) \}$.

For instance, given the relation CLINICAL_RECORD and the condition $t \supseteq \{1\}$ asking for valid times containing the chronon 1, $\sigma_{\mathrm{NEC}(t\supseteq\{1\})}{}^{\mathrm{TI}}$(CLINICAL_RECORD) = {(*Tim, breath* | {{1}, {1,2}, {1,3}, {1,2,3}}) }, while $\sigma_{\mathrm{POSS}(t\supseteq\{1\})}{}^{\mathrm{TI}}$(CLINICAL_RECORD) = CLINICAL_RECORD. We are not committed to any specific syntax for φ. Besides predicates asking for validity at (or before, or after) specific chronons, we also envision predicates about duration, and about the relative temporal location of tuples (based on Allen's relations) as in [21].

As the DTE set operators are used in the definition above, it is useful to consider some nice properties of the DTE set operators which have bearing on the relational algebraic operators.

Property 3 Closure of DTE set operators. The representation language of DTEs is closed with respect to the operations of $\cup^{\mathrm{DTE}}$, $\cap^{\mathrm{DTE}}$ and $-^{\mathrm{DTE}}$.

Our approach is a consistent extension of BCDM's one (considering valid time only).

Property 4 Consistent extension (DTEs). Determinate time is represented by singleton DTEs. If only singleton DTEs are used, the set operators $\cup^{\mathrm{DTE}}$, $\cap^{\mathrm{DTE}}$, and $-^{\mathrm{DTE}}$ are equivalent to the standard set operators ∪, ∩ and –, and the relational operators $\cup^{\mathrm{TI}}$, $-^{\mathrm{TI}}$, $\sigma_P{}^{\mathrm{TI}}$, $\sigma_\varphi{}^{t\ \mathrm{TI}}$, $\pi_X{}^{\mathrm{TI}}$ and $\times^{\mathrm{TI}}$ are equivalent to the standard BCDM valid-time relational operators $\cup^t$, $-^t$, $\sigma_P{}^t$, $\sigma_\varphi{}^t$, $\pi_X{}^t$ and $\times^t$.

3 Compact Representations

3.1 General methodology

The above treatment of valid-time indeterminacy is expressive but has several limitations. It is not compact and thus possibly not suitable [15] nor user-friendly, since all possible scenarios need to be elicited. More compact (and possibly more efficient) representations of temporal indeterminacy can be devised, sometimes at the price of losing part of the data expressiveness of the reference extensional approach. However, the limited expressiveness may be acceptable in several real-world domains. Instead of proposing a single compact representation, in this paper we explore (part of) the range of possibilities. Each possibility is characterized by a different way of representing in a compact way indeterminate temporal elements. On the other hand, it is worth stressing that, for all of our representations, we polymorphically apply:

i) the same way of defining tuples and relations;
ii) the same general definition of algebraic relational operators proposed in Definition 6.

Specifically, given a type *X* representation of the temporal component we subsequently define (there are several such representations we will consider), we adopt the following polymorphic definition of tuple and relation, an extension of Definition 4.

Definition 7 (Valid-time) indeterminate tuple and relation in a compact representation *X*. Given a schema $(A_1, \ldots, A_n)$ (where each A_i represents a non-temporal attribute on the domain D_i), let VT_X be the temporally indeterminate valid time attribute under representation *X*, let D_X be the domain of VT_X, and let a (valid-time) indeterminate relation *r* for the representation *X* be an instance of the schema $(A_1, \ldots, A_n \mid VT_X)$ defined over the domain $D_1 \times \ldots \times D_n \times D_X$ in which empty valid times and value-equivalent tuples are not admitted (as in BCDM). Each tuple $x = (v_1, \ldots, v_n \mid d_X) \in r$ is termed a (valid-time) indeterminate tuple for the representation *X*. Additionally, in all the cases, we always adopt the same definition of the algebraic relational operators (Definition 6), in which the union, intersection and difference operators between the temporal components have to be polymorphically instantiated with the specific operators defined for the type *X* of the temporal components.

As a consequence, in the following we focus only on the definition of representation formalisms for temporal components, and on the definition of intersection, union and difference set operators on temporal components. For each representation that we identify, we have adopted a uniform methodology:

i) we specify its extensional semantics by defining a function *Ext* that associates with a temporal component its extensional semantics represented as a DTE;
ii) we analyze its data expressiveness, both in terms of the reference approach, and with respect to the standard determinate approach;
iii) we define the intersection, union and difference set operators between temporal components, proving their correctness; and
iv) we ascertain the properties of the operators, and of the induced algebraic operators.

In particular, given a compact representation *X*, and given the set operations $\cup^X$, $\cap^X$, and $-^X$ on temporal components in *X*, as regards the data representation formalism (point (ii) above), we verify whether *X* is a ***consistent extension*** of the determinate temporal model, i.e., if *X* can express all the possible determinate temporal components. As regards the set operations, we consider the following properties:

- ***Closure***. The set operations $\cup^X$, $\cap^X$, and $-^X$ are closed (with respect to the representation *X*) if any application of the operations on temporal components in *X* provides as output a temporal component expressible in *X*.

- ***Correctness***. Temporal components in a representation *X* are compact representations of DTEs. Set operators $\cup^X$, $\cap^X$, and $-^X$ perform a "symbolic manipulation" on such representations, providing a compact representation as a result (i.e., the result is a temporal component in *X*). In other words, the result of any set operation $T_1^X\ Op^X\ T_2^X$ is a temporal component T_3^X in *X* which is directly computed only on the basis of the input (i.e., of $T_1^X\ Op^X\ T_2^X$) without resorting to their underlying semantics (i.e., to the DTEs $Ext(T_1^X)$ and $Ext(T_2^X)$). This procedure is efficient, since it only requires a symbolic manipulation on a compact representation, but demands a proof of correctness. Indeed, we have to prove the correctness of our set operators with respect to the extensional semantics: the symbolic manipulation provides the same results (expressed in the representation *X*) that would be obtained by operating on the corresponding extensions in the reference approach (i.e., by operating on DTEs). Formally speaking, we have to prove that, given a compact representation *X*, and any two temporal components T_1^X and T_2^X in *X*, we have that:

$$Ext(T_1^X \cup^X T_2^X)= Ext(T_1^X) \cup^{DTE} Ext(T_2^X)$$
$$Ext(T_1^X \cap^X T_2^X) = Ext(T_1^X) \cap^{DTE} Ext(T_2^X)$$
$$Ext(T_1^X -^X T_2^X) = Ext(T_1^X) -^{DTE} Ext(T_2^X).$$

- ***Consistent extension of set operators.*** For representations *X* that are a ***consistent extension*** of the determinate temporal model, set operators $\cup^X$, $\cap^X$, and $-^X$ are a consistent extension of the corresponding determinate-time set operators (e.g., of BCDM's operators $\cup^t$, $\cap^t$, and $-^t$) if, in case only temporal components T^{X}'s expressing determinate temporal components (in the representation *X*) are considered, $\cup^X$, $\cap^X$, and $-^X$ and $\cup^t$, $\cap^t$, and $-^t$ are equivalent.

- ***Consistent extension of the indeterminate relations and of the algebraic operators.*** Finally, given a compact representation *X*, tuples, relations and algebraic operations in *X* are polymorphically defined on the basis of temporal components T^X and set operations $\cup^X$, $\cap^X$, and $-^X$ in *X* (see Definition 7). Therefore, from the properties of consistent extension of the data model and of the set operators in a representation *X*, we can always induce that the relations and algebraic operations in *X* are a consistent extension of determinate (e.g., BCDM's) ones.

The range of possible representations has been identified by considering several different refinements. Our choice has been driven by considerations on expressiveness and usefulness derived from our previous research experience in both Temporal Databases and Artificial Intelligence, and in many applicative domain, ranging from medicine to geology. However, in no way do we claim that the refinements we have identified are the only ones worth investigating.

We begin with a basic and simple representation, in which temporal components only consist of independent indeterminate chronons. This basic representation is then successively refined into four additional, more expressive refined representations:

1. Possibility of expressing, besides indeterminate chronons, also a determinate component;
2. Possibility of coping with non-independent indeterminate chronons (i.e., capability of listing alternative sets of possibilities, possibly excluding some of the possible combinations);
3. Possibility of expressing a minimum constraint on the number of chronons;
4. Possibility of expressing a maximum constraint on the number of chronons.

Refinement 1 is important to model several domains (e.g., medicine) in which valid time is usually only partially unknown. This possibility is present in several models, both in Artificial Intelligence (consider, e.g., Allen [1]) and in TDB (e.g., Dyreson and Snodgrass [9]). Refinement 2 derives from the relevance of coping with alternatives in several domains (e.g., in planning), which is provided by many approaches, especially in Artificial Intelligence [1]. Refinements 3 and 4 support the treatment of minimal and maximal durations, as required in many domains (e.g., medicine).

The rest of this section is organized as follows. First, Section 3.2 discusses the "basic" compact representation. Then, in Sections 3.3-3.5, the basic representation is extended to cope with the above possibilities, independently of each other (for the sake of brevity, the possibility of expressing minimum and maximum constraints is considered together). Finally, in Section 3.6 the combination of all the different possibilities is taken into account.

3.2 Independent indeterminate chronons

In this section we present a compact representation useful in domains where one can identify a (possibly empty) set of chronons in which the fact may hold (*indeterminate chronons*), and such chronons are independent of each other, in the sense that all combinations of indeterminate chronons are possible alternative scenarios. For instance, consider the following.

Example 5. On Jan 1 2010 Ann might have had breathing problems between 1am (inclusive) and 4am (exclusive).

Here the fact may not hold, or it may hold in each of the hours 1, 2, and 3, considered independently of each other (meaning that it may hold at $\varnothing$, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}). In this section, we show that valid times of this type can be modeled by a representation formalism that is (strictly) less *data expressive* than the formalism of DTEs, yet supports a more compact and user-friendly representation.

Definition 9 Indeterminate temporal element, termed ITE. An ITE $<i>$ is represented by a temporal element, i.e., $i \subseteq TC$.

The extensional semantics of such a representation can be formalized taking advantage of the reference approach in Section 2.

Definition 10 Extensional semantics of ITEs. The semantics of an ITE $<i>$ is the DTE consisting of all and only the combinations of the chronons in i, i.e., $Ext(<i>)= PS(i)$.

Example 5 can be represented by the indeterminate temporal element {1,2,3}, and its underlying semantics is the DTE $Ext(<\{1,2,3\}>) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.[1]

ITEs are less expressive than DTEs, since not all combinations of temporal scenarios can be expressed.

Property 5 Expressiveness of ITE. Given a temporal domain TC, ITEs allow one to express all and only the elements of $PS(PS(TC))$ of the form $PS(INDET)$, where $INDET \subseteq TC$.

Intuitively, the formalism only allows one to cope with those subsets of TC in which all the possible combinations of indeterminate chronons are present. For instance, Example 3 is not expressible, since there is a dependency between the indeterminate chronons 1 and 2, which are mutually exclusive.

We now define the set operators on ITEs. In one sense, we have already done so, in Definition 5. However, that definition is in terms of the extension, whereas we would like to operate directly *at the level of the representation*, which is a succinct characterization of a set of scenarios, as expressed by the extension.

It turns out that the set operators are quite natural to express directly in the ITE representation.

Definition 11 Set operators $\cup^{ITE}$, $\cap^{ITE}$, and $-^{ITE}$ on ITEs. Given two ITEs $<i>$ and $<i'>$,

$$<i> \cup^{ITE} <i'> = < i \cup i'>$$
$$<i> \cap^{ITE} <i'> = < i \cap i'>$$
$$<i> -^{ITE} <i'> = <i>.$$

The union (intersection) of two ITEs is the ITE resulting from the union (intersection) of the sets of the chronons in the ITEs. Interestingly, the difference between two ITEs is the minuend. Specifically, the chronons in the ITEs are only *possible*, not *definite*, so that the chronons in the subtrahend may not exist, and so, they must not be subtracted from the indeterminate chronons in the minuend.

ITE tuples and relations can be polymorphically defined as shown by Definition 7. In particular, an ITE tuple is a non-temporal tuple paired with an ITE, and an ITE relation is a set of non-value equivalent ITE tuples. To define the relational temporal algebraic operators on ITE relations, we polymorphically adopt the definition of relational algebraic temporal operators of the extensional semantics (see Definition 6), in which the set operators $\cup^{DTE}$, $\cap^{DTE}$ and $-^{DTE}$ on DTEs are substituted by the set operators $\cup^{ITE}$, $\cap^{ITE}$ and $-^{ITE}$ on ITEs**.**

Property 6. Properties of the ITE representation. ITE set operators are closed and correct. No consistent extension property holds in ITE.

Proof. Correctness of intersection ($\cap^{ITE}$):

Since, by definition, $<i> \cap^{ITE} <i'> = < i \cap i'>$, we have to prove that $Ext(<i>) \cap^{DTE} Ext(<i'>) = Ext(< i \cap i'>)$. By the semantics of ITEs, $Ext(<i>)=PS(i)$ and, by the definition of intersection between DTEs and by the distributive law of intersection over power sets, $Ext(<i>) \cap^{DTE} Ext(<i'>) = PS(i) \cap^{DTE} PS(i') = \{ a \cap b \mid a \in PS(i) \text{ and } b \in PS(i') \} = PS(i \cap i') = Ext(<i \cap i'>)$. □

As regards the consistent extension property, let us consider the DTE {{1}}, containing just the determinate-time temporal element {1}: it is not possible to model it with an ITE because the extension of any ITE necessarily contains also the empty temporal element $\varnothing$.

A drawback of ITEs is that they represent only indeterminate chronons. Thus, ITEs cannot represent determinate time. An ITE can represent that Ann might have had breathing problems between 1am and 4am (Example 5), but not that Ann definitely had breathing problems at 5am. This limitation implies that ITE relations are not a consistent extension of BCDM, and ITE relational operators are not a consistent extension of BCDM operators. However, such properties will hold for the representation to be described in the following Section.

3.3 Determinate chronons

In this section we present a compact representation useful in domains where, besides independent indeterminate chronons, one can identify a (possibly empty) set of chronons in which the fact certainly holds (termed *determinate chronons*). For instance, consider Example 4 in Section 2.2.Valid times of this type can be modeled by a representation formalism that is (strictly) less *data expressive* than the formalism of DTEs, yet supports a more compact and user-friendly representation.

Definition 12 Determinate+Indeterminate temporal element, termed DITE. A DITE is a pair $<d,i>$, where d and i are temporal elements.

Intuitively, the first element of the pair identifies the

[1] Notice that, for the sake of efficiency, contiguous sets of chronons in each temporal element can be compactly represented by the periods covering them (e.g., {{1,2,3,4,6,7,8}, {8,9,10}} can be equivalently represented by {{[1-4],[6-8]},{[8-10]}}).

determinate chronons, and the second element the indeterminate ones. The extensional semantics of such a representation can be formalized taking advantage of the general approach in Section 2.

Definition 13 Extensional semantics of DITEs. The semantics of a DITE $<d,i>$ is the DTE consisting of all and only the sets that contain d and the combinations of the chronons in i, i.e., $Ext(<d,i>) = \{ d \cup e \mid e \subseteq i \}$.

Example 4 can be represented by the determinate+indeterminate temporal element <{1},{2,3}>, and its underlying semantics is the DTE $Ext(<\{1\},\{2,3\}>) = \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$.

DITEs are less expressive than DTEs, since not all combinations of temporal scenarios can be expressed.

Definition 14 Set operators $\cup^{DITE}$, $\cap^{DITE}$, and $-^{DITE}$. Given two DITEs $<d,i>$ and $<d',i'>$,

$<d,i> \cup^{DITE} <d',i'> = <d \cup d', i \cup i'>$

$<d,i> \cap^{DITE} <d',i'> = <d \cap d', (d \cup i) \cap (d' \cup i')>$

$<d,i> -^{DITE} <d',i'> = <d - (d' \cup i'), (d \cup i) - d'>$. ■

Property 7. Properties of the DITE representation. DITE set operators are closed and correct. The consistent extension properties hold in DITE.

A detailed treatment of DITEs, of the related algebra and of its properties is reported in the preliminary version of this work in [2].

3.4 Dependent indeterminate chronons

Coping with non-independent indeterminate chronons involves the necessity of preventing some combinations of indeterminate chronons from being included in the extensional semantics of the temporal components. Consider Example 3, where not all the combinations of the chronons are allowed because hours 1 and 2 are mutually exclusive. In this section, we augment the basic representation (which only considers independent indeterminate chronons) to model also dependent indeterminate chronons and we describe a representation formalism that is (strictly) less *data expressive* than the formalism of DTEs, yet more compact and user friendly.

Definition 15 Dependent Indeterminate temporal element, termed DeITE. A DeITE is a set $\{i_1, \ldots, i_n\}$, where each i_j is a temporal element, i.e., $i_j \subseteq TC$.

Intuitively, the semantics of a DeITE is the union of the semantics of the ITEs $i_1, \ldots, i_n$.

Definition 16 Extensional semantics of DeITEs. The semantics of a DeITE $\{i_1, \ldots, i_n\}$ is the DTE consisting of all and only the sets that contain the combinations of the chronons in each i_j, i.e., $Ext(\{i_1, \ldots, i_n\}) = \{ e \mid e \subseteq i_1 \vee \ldots \vee e \subseteq i_n \}$.

Example 3 can be represented by the dependent indeterminate temporal element {{1},{2}} and its underlying semantics is the DTE $Ext(\{\{1\},\{2\}\}) = \{\emptyset, \{1\}, \{2\}\}$.

DeITEs are less expressive than DTEs, since not all combinations of temporal scenarios can be expressed.

Property 8 Expressiveness of DeITE. Given a temporal domain TC, DeITEs allow one to express all and only the subsets of $PS(PS(TC))$ of the form $PS(INDET_1) \cup \ldots \cup PS(INDET_n)$, where $INDET_j \subseteq TC$, $j=1, \ldots, n$.

This property states that DeITEs are less expressive than DTEs, since not all combinations of temporal scenarios can be expressed. For instance, Example 1 cannot be expressed with a DeITE: in fact John certainly had breathing problems, so that the empty temporal element $\emptyset$ must not be in the extensional semantics of the DeITE, but with a DeITE it is not possible to exclude $\emptyset$.

Definition 17 Set operators $\cup^{DeITE}$, $\cap^{DeITE}$, and $-^{DeITE}$. Given two DeITEs $\{i_1, \ldots, i_n\}$ and $\{i'_1, \ldots, i'_h\}$,

$\{i_1, \ldots, i_n\} \cup^{DeITE} \{i'_1, \ldots, i'_h\} = \{i_j \cup i'_k \mid 1 \le j \le n, 1 \le k \le h\}$

$\{i_1, \ldots, i_n\} \cap^{DeITE} \{i'_1, \ldots, i'_h\} = \{i_j \cap i'_k \mid 1 \le j \le n, 1 \le k \le h\}$

$\{i_1, \ldots, i_n\} -^{DeITE} \{i'_1, \ldots, i'_h\} = \{i_1, \ldots, i_n\}$.

The union, intersection and difference between two DeITEs is the pairwise union, intersection and difference of the ITEs that compose the DeITEs (see the definition of $\cup^{ITE}$, $\cap^{ITE}$, and $-^{ITE}$).

The following properties hold for DeITE:

Property 9. Properties of the DeITE representation. DeITE set operators are closed and correct. No consistent extension property holds in DeITE.

As regards consistent extension, since ITEs are a special case of DeITEs with one component, the same counterexample provided for ITEs is applicable here.

3.5 Minimum and maximum cardinality

Minimum and maximum cardinality constraints are useful in order to explicitly model constraints about temporal duration. For instance, the constraint that ischemic stroke happened in at most one hour (see Example 2) can be stated by setting the maximum cardinality constraint to 1.

In this section, we augment the basic representation with independent indeterminate chronons to model minimum and maximum constraints on the components.

Definition 18 Independent Indeterminate temporal element with minimum/maximum constraints, termed mMITE. An mMITE is a triple $<i, m, M>$, where i is a temporal element, m and M are non-negative integers, specifying the minimum and maximum cardinalities, respectively, with $m \le M$.

Definition 19 Extensional semantics of mMITEs. The semantics of an mMITE $<i, m, M>$ is the DTE consisting of all and only the combinations of the chronons in i with cardinality between m and M, i.e., $Ext(<i, m, M>) = \{ e \mid e \subseteq i \wedge m \le |e| \le M \}$.

Consider the following example.

Example 6. On Jan 1 2010 between 2am (inclusive) and 5am (exclusive) Sue had breathing problems for two hours within that three-hour period.

Example 6 can be compactly represented by the mMITE <{2,3,4},2,2>, and its underlying semantics is the DTE $Ext(<\{2,3,4\},2,2>) = \{\{2,3\}, \{2,4\}, \{3,4\}\}$.

mMITEs are less expressive than DTEs, since not all combinations of temporal scenarios can be expressed.

Property 10 Expressiveness of mMITE. Given a temporal domain TC, a subset $INDET$ of TC and two non-negative integers m and M with $m \le M$, mMITEs allow one to express all and only the subsets of $PS(PS(TC))$ of the form $PS(INDET)$, whose cardinalities are between m and M.

Example 4 cannot be represented with a mMITE: in fact, if the component i of the mMITE has to contain the chronons 1, 2 and 3 (since Tim had breathing problems in such hours) and if the extension of the mMITE has to contain the chronon 1 alone, it must also contain all the other temporal elements with cardinality 1 (i.e., the chronons 2 and 3 alone), while they are not possible.

Unfortunately, this representation is not closed with regard to the set operators and, thus, also the relative relational algebra is not closed. For instance, we show that the difference set operator is not closed. In order to be correct, the mMITE difference set operator should satisfy:

$Ext(<i, m, M> -^{\mathrm{mMITE}} <i', m', M'>) = Ext(<i, m, M>) -^{\mathrm{DTE}} Ext(<i', m', M'>)$.

Let us consider $<\{1, 2, 3\},1,3> -^{\mathrm{mMITE}} <\{2,3\},1,2>$. If the difference is defined correctly (with respect to the reference approach), the result of the above operation must be $Ext(<\{1, 2, 3\},1,3>) -^{\mathrm{DTE}} Ext(\{2,3\},1,2>) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}\}$. However, this DTE is not expressible by an mMITE; in fact, the temporal element of cardinality 2 {2,3} is missing (see Property 10).

3.6 Combinations

We have explored all possible combinations of the above refinements (indeed, we have also considered the minimum and the maximum constraints as independent refinements, to be combined with the other ones). For the sake of brevity, in this section we only consider the representation that includes all the refinements: determinate and indeterminate chronons, dependent indeterminate chronons, and minimum and maximum cardinality. A systematic analysis of all the representations we explored is given in the next section.

Definition 20 Determinate+Dependent Indeterminate temporal element with minimum/maximum cardinality, termed mMDDeITE. An mMDDeITE is a pair $<d, \{<i_1,m_1,M_1>, \dots, <i_n,m_n,M_n>\}>$, where d is a temporal element, and for j=1, …, n i_j are temporal elements, m_j and M_j are non-negative integers, and $m_j \le M_j$.

Definition 21 Extensional semantics of mMDDeITEs. The semantics of a mMDDeITE $<d, \{<i_1,m_1,M_1>, \dots, <i_n,m_n,M_n>\}>$ is the DTE consisting of all and only the sets that contain the chronons in d and the combinations of the chronons in each i_j that satisfy the cardinality constraint, i.e., $Ext(<d, \{<i_1,m_1,M_1>, \dots, <i_n,m_n,M_n>\}>) = \{ d \cup e \mid (e \subseteq i_1 \wedge m_1 \le |e| \le M_1) \vee \dots \vee (e \subseteq i_n \wedge m_n \le |e| \le M_n) \}$.

Consider the following example.

Example 7. On Jan 1 2010 Ann-Marie had breathing problems at 1am, and then either for 1–2 hours between 3am (inclusive) and 6am (exclusive) or for 1–2 hours between 8am (inclusive) and 10am (exclusive).

Example 7 can be represented by the mMDDeITE <{1}, {<{3,4,5},1,2>, <{8,9},1,2>}> and its underlying semantics is the DTE Ext(<{1}, {<{3,4,5},1,2>, <{8,9},1,2>}>) = {{1,3}, {1,4}, {1,5}, {1,3,4}, {1,3,5}, {1,4,5}, {1,8}, {1,9}, {1,8,9}}.

mMDDeITEs are as expressive as DTEs, thus all combinations of temporal scenarios can be expressed.

Property 11 Expressiveness of mMDDeITE. Given a temporal domain TC, mMDDeITEs allow one to express all and only the subsets of $PS(PS(TC))$.

In other words, mMDDeITEs have the same expressiveness of DTEs, that is, of the full extension. Intuitively, given a DTE $dte = \{\{c_{h1}, \dots, c_{hk}\}, \dots, \{c_{i1}, \dots, c_{il}\}\}$, it is possible to define a mMDDeITE having dte as an extension by setting each first component i_j (j=1, …, n) of the triplets $<i_j, m_j, M_j>$ of the mMDDeITE to one of the elements of dte, i.e., the mMDDeITE corresponding to dte is $<\emptyset, \{<\{c_{h1}, \dots, c_{hk}\},k,k>, \dots, <\{c_{i1}, \dots, c_{il}\},l,l>\}>$.

Determinate valid time can be easily captured by means of mMDDeITEs.

At this point, the set operations of union ($\cup^{\mathrm{mMDDeITE}}$), intersection ($\cap^{\mathrm{mMDDeITE}}$) and difference ($-^{\mathrm{mMDDeITE}}$) between mMDDeITEs can be defined.

Definition 22 $\cup^{\mathrm{mMDDeITE}}$, $\cap^{\mathrm{mMDDeITE}}$, and $-^{\mathrm{mMDDeITE}}$. Given two mMDDeITEs $<d, \{<i_1,m_1,M_1>, \dots, <i_n,m_n,M_n>\}>$ and $<d', \{<i'_1,m'_1,M'_1>, \dots, <i'_h,m'_h,M'_h>\}>$,

$<d, \{<i_1,m_1,M_1>, \dots, <i_n,m_n,M_n>\}> \cup^{\mathrm{mMDDeITE}} <d', \{<i'_1,m'_1,M'_1>, \dots, <i'_h,m'_h,M'_h>\}> =$

$<d \cup d', \{ <a \cup b, |a \cup b|, |a \cup b|> \mid a \subseteq i_j, b \subseteq i'_k \mid m_j \le |a| \le M_j, m_k \le |b| \le M_k \mid j=1, \dots, n, k=1, \dots, h \}>$

$<d, \{<i_1,m_1,M_1>, \dots, <i_n,m_n,M_n>\}> \cap^{\mathrm{mMDDeITE}} <d', \{<i'_1,m'_1,M'_1>, \dots, <i'_h,m'_h,M'_h>\}> = <d \cap d', \{<(d \cup a) \cap (d' \cup b), |(d \cup a) \cap (d' \cup b)|, |(d \cup a) \cap (d' \cup b)|> \mid a \subseteq i_j, b \subseteq i'_k \mid m_j \le |a| \le M_j, m_k \le |b| \le M_k \mid j=1, \dots, n, k=1, \dots, h \} >$

$<d, \{<i_1,m_1,M_1>, \dots, <i_n,m_n,M_n>\}> -^{\mathrm{mMDDeITE}} <d', \{<i'_1,m'_1,M'_1>, \dots, <i'_h,m'_h,M'_h>\}> = <d - (d' \cup i'_1 \cup \dots \cup i'_h), \{ <(d \cup a) - (d' \cup b), |(d \cup a) - (d' \cup b)|, |(d \cup a) - (d' \cup b)|> \mid a \subseteq i_j, b \subseteq i'_k \mid m_j \le |a| \le M_j, m_k \le |b| \le M_k \mid j=1, \dots, n, k=1, \dots, h\}>$.

The definition of the mMDDeITE operators generalizes the operators described in the previous sections. The determinate component of the output is evaluated as for the DITE [2] (obviously, for the determinate component of the difference, we exclude all the ITEs in the indeterminate component of the subtrahend).

For the indeterminate component, we consider the subsets $a \subseteq i_j$, $b \subseteq i'_k$ of the input indeterminate components that satisfy the minimum and maximum constraints, and we perform pairwise union, intersection and difference (see the definition of the DeITE operators in Section 3.4) by considering also the determinate component (see the definition of the DITE operators in Section 3.2). The minimum/maximum cardinalities are the cardinalities of the resulting sets.

Property 12. Properties of the mMDDeITE representation. mMDDeITE set operators are closed and correct. The consistent extension properties hold in mMDDeITE.

4 Comparison of the Representations

Since the four refinements pointed out in Section 3.1 are orthogonal, implying that all possible combinations are feasible, in our overall approach we have identified sixteen different languages to express valid-time indeterminacy, plus the extensional one discussed in Section 2. We have considered five out of the sixteen languages in Sections 3.2–3.6. In this section, we provide a general overview of the whole family of representations, analyzing and comparing them.

Notation. In the following, we use short tags to denote the refinements and, then, the seventeen formalisms. RA denotes the reference approach introduced in Section 2. I

denotes the treatment of indeterminate chronons, and *D+I* the treatment of both determinate and indeterminate chronons. Superscript * denotes the possibility of specifying multiple alternatives concerning the indeterminate temporal element (i.e., of coping with non-independent indeterminate chronons). Finally, superscripts *n* and *N* denote the possibility of expressing minimality and maximality constraints respectively. Combinations of tags represent combinations of refinements.

The seventeen languages thus are *RA*, *I*, *D+I*, I^*, $D+I^*$, I^n, $D+I^n$, $I^{n,*}$, $D+I^{n,*}$, I^N, $D+I^N$, $I^{N,*}$, $D+I^{N,*}$, $I^{n,N}$, $D+I^{n,N}$, $I^{n,N,*}$, *and* $D+I^{n,N,*}$. Specifically, *I*, *D+I*, I^*, $I^{n,N}$, $D+I^{n,N,*}$ correspond to the representations discussed through Sections 3.2–3.6. In the following, we discuss important properties that some of the languages share.

4.1 Closure

The first, fundamental property we consider is closure. In fact, if the temporal representation is not closed with respect to the set operators of union, intersection and difference, the relational algebra itself (defined in Section 2.4) is not closed. Hence, representations for which closure does not hold are not suitable in the DB context.

Property 13 Closure. The formalisms *RA*, *I*, *D+I*, I^*, $I^{n,*}$, $D+I^{n,*}$, $I^{N,*}$, $D+I^{N,*}$, $I^{n,N,*}$, $D+I^{n,N,*}$ are closed with respect to set operators, while the formalisms $D+I^*$, I^n, $D+I^n$, I^N, $D+I^N$, $I^{n,N}$, $D+I^{n,N}$ are not.

We have shown in Section 3.5 that $I^{n,N}$ is not closed. In general, we can see that the addition of the minimal and/or maximal constraint, if it is not paired with the possibility of specifying multiple alternatives concerning the indeterminate temporal element (* symbol), leads to representation languages that are not closed, independently of whether the treatment of determinate chronons is considered. Intuitively, this is because, considering the lattice of scenarios introduced in Section 2.3, I^n, I^N , $I^{n,N}$, $D+I^n$, $D+I^N$, $D+I^{n,N}$ can represent the entire part of the lattice from which we possibly exclude a bottom part (because of the minimum cardinality) and/or a top part (because of the maximum cardinality). However, the difference between two mMITEs can generate a region not definable by simply cutting away a bottom or top part of the lattice (see the counterexample in Section 3.5).

Moreover, it is interesting to notice that, even though the language *D+I*, described in Section 3.3, is closed, adding the possibility of listing alternatives concerning indeterminate chronons (i.e., the language $D+I^*$) results in a language that is not closed. For example, consider the set operator of difference and the operation in $D+I^*$ <{1,2}, ∅> – <∅, {{1}, {2}}>. The extensional semantics of the result is the DTE {{1}, {2}, {1,2}}, which is not expressible in $D+I^*$ since it has an empty determinate component (because the temporal elements have no common chronon), but the empty temporal element is not present (and a DeITE cannot exclude ∅ when the determinate component is empty).

In the remainder of this section, we further investigate the properties of the closed representations.

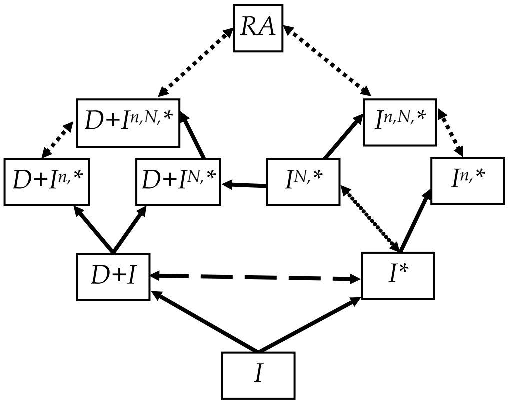

Figure 2. Graphical representation of the data expressiveness of the nine closed representations for expressing valid-time indeterminacy studied in our work (as well as the reference approach, *RA*).

4.2 Expressiveness

Considering the closed representations, we compare their expressiveness in Figure 2. In this figure, the representations are denoted as rectangles. Solid arcs connect a less expressive to a more expressive language. Dotted arcs connect languages with equal expressiveness. The dashed arc connects two incomparable languages. The relations derivable by transitive closure are not represented.

We have proven that four of the nine closed representations are as expressive as the reference approach *RA*.

Property 14 Expressiveness. The representations $D+I^{n,*}$, $D+I^{n,N,*}$, $I^{n,*}$, $I^{n,N,*}$ are as expressive as *RA*. *I*, *D+I*, I^*, $I^{N,*}$, $D+I^{N,*}$ are less expressive than *RA*.

In general, the possibility of setting a minimum constraint, in addition to the possibility of specifying multiple alternatives concerning the indeterminate temporal element (i.e., * plus *n*), renders a language as expressive as *RA* i.e., such that any DTE *X* can be represented by the formalisms. Intuitively, this is because through the alternative refinement (* feature) one can elicit all temporal elements in *X*. In principle, the extensional semantics of each alternative is not just one temporal element, but the power set of the chronons it contains. However, by imposing for each alternative the constraint that the minimum constraint must be exactly the number of chronons in that alternative, just all and only the sets that are the temporal elements in *X* are considered.

Thus, $D+I^{n,*}$, $D+I^{n,N,*}$, $I^{n,*}$, $I^{n,N,*}$ can express (possibly in a more compact way) all the possible combinations of alternative scenarios.

It is interesting to notice how the expressiveness changes as we add refinements to a language. For example, starting from the *D+I* representation, if we add the possibility of expressing alternatives concerning the indeterminate component, we derive the representation $D+I^*$, which is not closed, as commented above. However, if we add to *D+I* the possibility of expressing both alternatives concerning the indeterminate component and minimality constraints (refinements (2) and (3) in Section 3.1), we obtain a closed language, $D+I^{n,*}$, which is strictly more expressive, and that is as expressive as *RA*. If we add to

$D+I^*$ the possibility to express maximality constraints (obtaining $D+I^{N,*}$), we obtain a closed language, with different expressive power. In fact, $D+I^{N,*}$ cannot express arbitrary DTEs since all extensions have to include either the empty temporal element (since the determinate component is empty) or a same temporal element (since the determinate component is not empty).

On the other hand, starting from the I^* representation, if we add the possibility of expressing minimality constraints, we augment its expressivity resulting in a representation that is as expressive as *RA* (see the discussion above); however, if we add to I^* the possibility of expressing maximality constraints, the expressivity of the representation does not change. Indeed, given a set of chronons with maximum cardinality N, it can be equivalently represented by alternative sets of chronons. For instance, a set {1,2,3} with maximum cardinality 2 (whose extension is {∅, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}}) may be represented by the DeITE {{1,2}, {1,3}, {2,3}}.

An asymmetry in Figure 2 can be observed in that the expressiveness of $D+I$ cannot be compared with I^*. For example, on the one hand the DTE {∅, {2}, {3}} can be expressed by I^* as the set {{2},{3}} containing two alternatives, but cannot be expressed by $D+I$, because, since the empty temporal element is present, the determinate component must be empty, but including the chronons 2 and 3 in the indeterminate component would necessarily include also the temporal element {2,3}. On the other hand, we cannot conclude that I^* is more expressive than $D+I$, because, for example, the DTE {{2}, {2,3}} is expressible by $D+I$ as <{2},{3}>, but cannot be expressed by I^* because it does not contain the empty temporal element, which is necessarily contained in every DTE generated by I^*.

4.3 Consistent extension

The property of consistent extension (of BCDM) is also important, to grant for the compatibility and interoperability with existent BCDM-based representations.

Property 15 Consistent extension. The representations $D+I$, $D+I^{n,*}$, $D+I^{N,*}$, $D+I^{n,N,*}$, $I^{n,*}$, $I^{n,N,*}$, and *RA* are a consistent extension of BCDM. I, I^*, $I^{N,*}$ are not.

Of course, all the representations that have a determinate component are trivially a consistent extension of BCDM, since the determinate component models determinate BCDM times. And, trivially, *RA* models determinate time through singleton DTEs. Moreover, it is worth noticing that, while the representation I (i.e., independent indeterminate chronons, discussed in Section 3.2) is not a consistent extension, the addition of the possibility of expressing alternatives (*) and minimality constraint (n) to it grants the property. This is because—as discussed above—$I^{n,*}$ is as expressive as *RA* and thus it can model determinate time as *RA* does. On the other hand, I^* and $I^{N,*}$ are not consistent extensions of BCDM because they can represent only DTEs where the empty temporal element is necessarily present.

4.4 Existential indeterminacy

Another relevant property about expressiveness regards how the different representations cope with the indeterminacy about the existence of a given tuple (termed *existential indeterminacy*). All the representations allow to state that the fact described by the tuple may also not occur (notice that this fact can be represented in *RA* by including the empty set in the DTE; additionally, the empty set is necessarily included in the extensions of every ITE). On the other hand, not all the representations allow one to model the fact that there is no existential indeterminacy, i.e., that the tuple certainly exists (although we might not know exactly when).

Property 16 Existential indeterminacy. All the representations can represent existential indeterminacy. On the other hand, I, I^*, and $I^{N,*}$ cannot represent certainty of existence.

Of course, certainty of existence can be trivially represented by all representations that support determinate chronons. Similarly, the representations that do not provide certainty of existence cannot represent determinate time and, thus, are not consistent extensions of BCDM. Additionally, the possibility of specifying a minimum cardinality allows one to express certainty of existence, since the minimum cardinality allows one to exclude the empty set from the extensions.

4.5 Compactness and suitability (base relations)

Finally, it is worth stressing that expressiveness is not the only criterion worth to be considered when evaluating representations (otherwise *RA* could suffice). Compactness is also important, as is *suitability* [15]. For instance, consider Example 4: it can be expressed in a more compact way in $D+I$ than in $I^{n,N,*}$, even though $D+I$ is strictly less expressive than $I^{n,N,*}$. In fact, on the one hand in $D+I$ it can be expressed —as described in Section 3.3— as <{1},{2,3}>. On the other hand, in $I^{n,N,*}$ it can be expressed as the set of alternatives { <{1},1,1>, <{1,2},2,2>, <{1,3},2,2>, <{1,2,3},3,3> }, containing four alternatives.

As another example, consider:

Example 8. On Jan 1 2010 Tom might have had fever between 1am (inclusive) and 4am (exclusive) for at most 2 hours.

This example can be expressed in a more compact way in $I^{N,*}$ than in $D+I^{n,*}$, even though $I^{N,*}$ is strictly less expressive than $D+I^{n,*}$. In fact, in $I^{N,*}$ it can be expressed as {<{1,2,3},2>}, while in $D+I^{n,*}$ it can be expressed as <∅, {<{1,2},0>, <{1,3},0>, <{2,3},0>}>.

4.6 Evaluation of set operators

Until now we have considered, besides closure (which is required for making queries possible), properties related to the expressiveness of the representations, and their capability to cope with certain phenomena (possibly, in a suitable way). However, such properties have a cost, both in terms of the storage needed to represent (temporal) data, and in term of the (temporal) complexity of performing algebraic operators. Note that in order to have the closure property the minimum and/or maximum cardinality refinements cannot come alone, but require that also the "*" (multiple alternatives) refinement is provided.

Several factors can be considered to characterize the "cost" of refinements. In the following, we consider the length of the output of set operators on the different types of temporal components, which, besides storage requirements, also gives an insight about the complexity needed to evaluate algebraic operations. The evaluation of set operators (and thus of algebraic operators) for the representations *I* and *D+I* (i.e., for ITEs and DITEs) simply involves union, intersection and difference on sets of chronons, and the length of the output is linear with respect to the length of the input. The introduction of the "*" refinement demands for a pairwise combination of the input alternatives for the evaluation of set operators, implying that the length of the output may be quadratic with respect to the length of the input. The introduction of the cardinality refinements further increases the complexity: by definition, all the subsets satisfying the cardinality constraints of the input sets of chronons must be taken into account. Thus, the output may grow exponentially with respect to the length of the input.

4.7 Summary

To wrap up, Table 1 compares along various aspects considered in this paper the ten representations that are closed with regard to set operators. The first four columns show the four refinements we identified in Section 3.1. Each column states whether it is possible to express a phenomenon in a compact/user-friendly way (e.g., *RA* allows one to express the minimal duration constraint, but only eliciting all possible cases; thus *RA* does *not* exhibit such a property). *Det* stands for the possibility of expressing determinate chronons, *Dep* for coping with dependent chronons, *Min* and *Max* for the possibility of expressing minimum and maximum constraints respectively. The fifth column focuses on the possibility of coping with certainty of existence; the sixth column takes into account expressiveness (only the representations that have the full expressiveness of *RA* are marked); the seventh column considers the consistent extension property[2]; Finally, the eighth column represents the cost of each representation *X*, considering the length of the output of set operators with respect to the length of their input (as expressed in the representation *X*). Considering cost, it is worth noticing that (i) *RA* is the most "costly" approach (even if its output is at most quadratic with respect to the input). This is due to the fact that the representations (of the input and of the output) are not compact: all the scenarios are explicitly represented. Thus, for instance, the evaluation of union must consider all possible pairs of scenarios, which is the upper bound for the complexity for all the representations (provided that they are correct with respect to *RA*), and (ii) all set operators of representations considering the "D" refinement have been defined in such a way that, if only determinate chronons are used, no additional cost is incurred with respect to standard approaches to determinate time.

TABLE 1.
COMPARISON OF THE TEN APPROACHES.

	Det	*Dep*	*Min*	*Max*	*Cert exist*	*Full Expr*	*Consist Ext*	*Size outp*
I								lin
I^*		X						quad
$I^{n,*}$		X	X		X	X	X	exp
$I^{N,*}$		X		X				exp
$I^{n,N,*}$		X	X	X	X	X	X	exp
$D+I$	X				X		X	lin
$D+I^{n,*}$	X	X	X		X	X	X	exp
$D+I^{N,*}$	X	X		X	X		X	exp
$D+I^{n,N,*}$	X	X	X	X	X	X	X	exp
RA		X			X	X	X	quad

5 PROBABILISTIC EXTENSION

In some approaches in the literature, temporal indeterminacy has been dealt with in conjunction to probabilities [9] [7]. Intuitively, probabilities, when available, provide additional pieces of information for discriminating between alternative scenarios. In the following, we sketch how our approach can be extended to cope with probabilities. We operate in two steps. First, we extend the reference approach to cope with probabilities. Then, we move towards compact representations. In particular, we only consider the formalism for independent indeterminate chronons (Section 3.2). The same methodology can be used to extend also the other representations. However, several challenging issues have to be taken into account, left for future work.

5.1 Probabilistic Reference approach

We assume that facts in the database are independent, and that for each fact temporal scenarios are exhaustive and mutually exclusive. For each fact in the database, we introduce a probability distribution function *P*, which gives the probability that the fact occurred in a scenario (i.e., in a temporal element associated with the fact).

Definition 23 Probabilistic disjunctive temporal element, termed PDTE. A probabilistic disjunctive temporal element is a disjunctive *S* set of temporal elements associated with a probability distribution function P: $S \rightarrow [0,1]$.

Notation. For the sake of simplicity, we annotate each temporal element with its probability and we term it as *probabilistic* temporal element.

Example 9. (*Sue*, *stroke* | $\{\emptyset^{0.4}, \{1\}^{0.1}, \{2\}^{0.4}, \{1,2\}^{0.1}\}$) represents the fact that on Jan 1 2010 Sue might have had an ischemic stroke either at 1am (with probability 0.1) or at 2am (with probability 0.4) or from 1am to 2am included (with probability 0.1) or might not (with probability 0.4).

As we did for DTEs, we define the (generalized) set operators of intersection, union and difference applied to PDTEs.

Definition 24 $\cup^{PDTE}$, $\cap^{PDTE}$, and $-^{PDTE}$. Given two PDTEs *DA* and *DB*, and denoting their probabilistic temporal elements by A^p and $B^{p'}$ respectively, the operations

[2] In Table 1, *Cert exist* and *Consist Ext* coincide. However, this is not a general rule. For instance, a formalism providing for enumerators such as "one_of" or "at_least_one" associated with temporal elements allows one to express certainty of existence, but it is not a consistent extension of BCDM (since purely determinate time cannot be represented).

Op^{PDTE} of union ($\cup^{\mathrm{PDTE}}$), intersection ($\cap^{\mathrm{PDTE}}$), and difference ($-^{\mathrm{PDTE}}$) between DA and DB are defined as the PDTE obtained through the pairwise application of standard set operations Op on A and B; the probability is the product $p*p'$ of the probabilities of A^p and $B^{p'}$. In the case that more than one pair of probabilistic temporal elements A^p and $B^{p'}$ gives rise to the same probabilistic temporal element $C^{p''}$, we sum all their products $p*p'$:

$DA\ Op^{\mathrm{PDTE}}\ DB = \{C^{p''} \mid \exists A^p \in DA\ \exists B^{p'} \in DB\ (C = A\ Op\ B) \wedge p'' = \Sigma_{A^p \in DA \wedge B^{p'} \in DB \wedge C=(A\ Op\ B)}\ p*p'\}$.

Example 10. $\{\varnothing^{0.4}, \{1\}^{0.1}, \{2\}^{0.4}, \{1,2\}^{0.1}\} \cap^{\mathbf{PDTE}} \{\varnothing^{0.3}, \{2\}^{0.2}, \{3\}^{0.3}, \{2,3\}^{0.2}\} = \{\varnothing^{0.8}, \{2\}^{0.2}\}$.

5.2 Probabilistic independent indeterminate chronons

In the compact representation I^P, as in I, we associate a set of chronons with a tuple. Since, as in I, there is no explicit representation of scenarios, in I^P we associate probabilities with each chronon.

Definition 25 Probabilistic indeterminate temporal element, termed PITE. A PITE $<i>$ is represented by a temporal element, i.e., $i \subseteq TC$, and a probability function $P^I: i \rightarrow (0,1]$.

Notice that, for the sake of compactness, we do not admit chronons with null probability in PITEs.

Notation. When there is ambiguity, we use the notation $P^I{}_i(c)$ to represent the probability of the chronon c in the PITE $<i>$.

Example 11. $<1^{0.2}, 2^{0.5}>$ represents that the fact holds in the hour 1am with probability 0.2, and in the hour 2am with probability 0.5. Notice that probabilities in a PITE do not necessarily sum up to 1, since they represent marginal probabilities with respect to the probabilities of the corresponding PDTEs (see Definition 26 below).

Definition 26 Extensional semantics of PITEs (Ext^P function). The semantics of a PITE $=<c_1{}^{p1}, \ldots, c_k{}^{pk}>$ is the PDTE consisting of all and only the probabilistic temporal elements resulting from the combinations of the chronons in $\{c_1, \ldots, c_k\}$; the probability of a probabilistic temporal element is the product of the probabilities that each chronon is or is not in the scenario, i.e.,

$Ext^P(<c_1{}^{p1}, \ldots, c_k{}^{pk}>) = \{\{c_i, \ldots, c_j\}^p \mid \{c_i, \ldots, c_j\} \subseteq \{c_1, \ldots, c_k\} \wedge p = p'^1 * \ldots * p'^k$, where $p'^l = p^l$ if $c_l \in \{c_i, \ldots, c_j\}$, $p'^l = (1-p^l)$ if $c_l \notin \{c_i, \ldots, c_j\}$.

Example 12. $Ext^P(<1^{0.2}, 2^{0.5}>) = \{\varnothing^{0.4}, \{1\}^{0.1}, \{2\}^{0.4}, \{1,2\}^{0.1}\}$, i.e., $<1^{0.2}, 2^{0.5}>$ is the compact PITE representation of the PDTE in Example 9 above.

For the sake of simplicity, in the following formulas we assume that, given a PITE i, if $c \notin i$ then $P^I{}_i(c)=0$.

Definition 27 Set operators $\cup^{\mathbf{PITE}}$, $\cap^{\mathbf{PITE}}$, and $-^{\mathbf{PITE}}$ on PITEs. Given two PITEs $<i_1>$ and $<i_2>$,

$<i_1> \cup^{\mathrm{PITE}} <i_2> = <\{c^p \mid (c \in i_1 \vee c \in i_2) \wedge p = P^I{}_{i1}(\mathrm{c}) * P^I{}_{i2}(\mathrm{c}) + P^I{}_{i1}(c) * (1 - P^I{}_{i2}(c)) + (1 - P^I{}_{i1}(c)) * P^I{}_{i2}(c)\}>$

$<i_1> \cap^{\mathrm{PITE}} <i_2> = <\{c^p \mid c \in i_1 \wedge c \in i_2 \wedge p = P^I{}_{i1}(c) * P^I{}_{i2}(c)\}>$

$<i_1> -^{\mathrm{PITE}} <i_2> = <\{c^p \mid c \in i_1 \wedge p = P^I{}_{i1}(c) * (1 - P^I{}_{i2}(c)) \wedge p \neq 0\}>$.

For intersection, we compute the set intersection of the chronons; the probability of each chronon in the result is the product of the input probabilities of the chronon in each set. For union, we compute the set union of the chronons; the probability of each chronon in the result is the sum of the probabilities that the chronon is in both the sets i_1 and i_2 or only in the set i_1 or only in the set i_2. For difference, the result is the minuend; the probability of each chronon is the probability that the chronon is in the minuend and is not in the subtrahend. If a chronon has null probability, it is not included in the result.

Example 13. $<1^{0.2}, 2^{0.5}> \cap^{\mathbf{PITE}} <2^{0.4}, 3^{0.5}> = <2^{0.2}>$. Notice that $Ext^P(<2^{0.4}, 3^{0.5}>) = \{\varnothing^{0.3}, \{2\}^{0.2}, \{3\}^{0.3}, \{2,3\}^{0.2}\}$, and $Ext^P(<2^{0.2}>) = \{\varnothing^{0.8}, \{2\}^{0.2}\}$, so that the above PITE intersection corresponds to the PDTE intersection in Example 10 (and is, indeed, *correct*).

The following property grants that the direct operations on PITEs are closed and correct with respect to the probabilistic reference approach PDTE. Notice that I^P is a consistent extension of BCDM since determinate chronons can be represented by associating them with the probability 1.

Property 17. Properties of the PITE representation. PITE set operators are closed and correct. PITE is a consistent extension of BCDM.

Proof. Correctness of intersection ($\cap^{\mathrm{PITE}}$):

We have to prove that $Ext^P(<i_1> \cap^{\mathrm{PITE}} <i_2>) = Ext^P(<i_1>) \cap^{\mathrm{PDTE}} Ext^P(<i_2>)$.

The definition of $\cap^{\mathrm{PITE}}$ consists of two parts, the former defining the output chronons, and the latter defining their probabilities. The first part of the definition is exactly the same as for ITEs, so that its proof of correctness has been already given. Let $i_1 = <\{c_1{}^{p1}, \ldots, c_l{}^{pl}, c'_1{}^{p'1}, \ldots, c'_m{}^{p'm}\}>$ and $i_2 = <\{c_1{}^{q1}, \ldots, c_l{}^{ql}, c''_1{}^{q''1}, \ldots, c''_n{}^{q''n}\}>$, where i_1 and i_2 have the common chronons $c_1, \ldots, c_l$. We thus have that $<\{c_1{}^{p1}, \ldots, c_l{}^{pl}, c'_1{}^{p'1}, \ldots, c'_m{}^{p'm}\}> \cap^{\mathrm{PITE}} <\{c_1{}^{q1}, \ldots, c_l{}^{ql}, c''_1{}^{q'1}, \ldots, c''_n{}^{q'n}\}> = <\{c_1{}^{r1}, \ldots, c_l{}^{rl}\}>$ is correct for some probability values $r_1, \ldots, r_l$. Now we have just to prove that $r_1 = p_1 * q_1, \ldots, r_l = p_l * q_l$.

Let us consider an arbitrary chronon $c_j \in \{c_1, \ldots, c_l\}$.

First we notice that, for the semantics of I^P (see the definition of Ext^P), $P^I{}_i(c_j)$ is the marginal probability of c_j in the probability distribution P of $Ext^P(<i>)$, i.e., $P^I{}_i(c_j) = \Sigma_{K^p \in Ext^P(<i>)\ \mid\ c_j \in K}\ p$.

Let $DC_{cj} \in (Ext^P(<i_1>) \cap^{\mathrm{PDTE}} Ext^P(<i_2>))$ be the subset of $Ext^P(<i_1>) \cap^{\mathrm{PDTE}} Ext^P(<i_2>)$ which contains only the probabilistic temporal elements including the chronon c_j. Let $DC_{cj} = \{ C_1{}^{s1}, \ldots, C_o{}^{so}\}$. Then, for the definition of intersection $\cap^{\mathrm{PDTE}}$, each $C_1{}^{s1}, \ldots, C_o{}^{so}$ is the intersection between a probabilistic temporal element A^p of $Ext^P(<i_1>)$ which contains c_j and a probabilistic temporal element $B^{p'}$ of $Ext^P(<i_2>)$ which contains c_j (i.e., $DC_{cj} = \{C^{p''} \mid \exists A^p \in Ext^P(<i_1>), \exists B^{p'} \in Ext^P(<i_2>)\ (c_j \in A \wedge c_j \in B \wedge C = A \cap B) \wedge p'' = \Sigma_{A^p \in Ext^P(<i1>) \wedge B^{p'} \in Ext^P(<i2>) \wedge c_j \in A \wedge c_j \in B \wedge C = A \cap B}\ p*p'\}$).

Thus, the probability r_j of the chronon c_j is the marginal probability of c_j, i.e., $r_j = \Sigma_{C^p \in DC_{cj}}\ p = ((1-p_1) * \ldots * p_j * \ldots * (1-p_l) * (1-p'_1) * \ldots * (1-p'_m)) * ((1-q_1) * \ldots * q_j * \ldots * (1-q_l) * (1-\mathrm{q}'_1) * \ldots * (1-q'_n)) + \ldots + (p_1 * \ldots * p_j * \ldots * p_l * p'_1 * \ldots * p'_m) * (q_1 * \ldots * q_j * \ldots * q_l * q'_1 * \ldots * q'_n) = p_j * q_j$. □

6 Related Work

In general, *temporal logics* have been extensively used for representing and reasoning about propositions and predicates whose truth depends on time. These systems are usually developed around a set of temporal connectives, such as *sometimes/always* in the future, *until* etc. that provide implicit reference to time instants. *First-order temporal logic* is a variant of temporal logic that allows first-order predicate symbols, variables and quantifiers, in addition to connectives. Many temporal logics have been proposed, differing in terms of expressiveness, order, time metric, temporal modalities, time model, and time structure (see, e.g., the survey in Emerson [10]). In the area of databases, some of such logics have been used as *temporal query languages* for timestamped temporal data (see, e.g., the survey by Chomicki and Toman – "Temporal Logic in Database Query Languages" entry in Liu and Tamer Özsu [19]).

Probabilistic temporal logics have been developed to reason about dynamic systems which include uncertainty and probabilistic assumptions. Both classical and non-classical logics have been extended to cope with probabilities. For instance, PCTL extends the branching time temporal logic CTL and is interpreted over discrete-time Markov chains; PTCTL extends the real-time branching logic TCTL, PDC extends the duration calculus DC; PNL extends the Neighbourhood Logic; Generalised Probabilistic Logic (GPL) is a Mu-calculus-based modal logic (references to these and other logics can be found in the recent survey by Konur [16]).

One of the earliest efforts to incorporate probabilistic information within a relational database is due to Cavallo and Pittarelli [4], who also proposed a partial relational algebra for the extended model. Probabilistic approaches have been widely used to cope with probabilistic temporal data and temporal indeterminacy (see, e.g., the recent survey "Probabilistic Temporal Databases" entry in Liu and Tamer Özsu [19]). For instance, Dekhtyar et al. [7] introduce temporal probabilistic tuples to cope with data such as "data tuple *d* is in relation *r* at some point of time in the interval $[t_i,t_j]$ with probability between *p* and *p'*." They also provide algebraic relational operators for their data model. However, they restrict their attention to events that are instantaneous, while our approach also considers events with duration (indeed, the minimum and maximum duration constraints would be meaningless with instantaneous events only). Another influent probabilistic approach to temporal indeterminacy has been proposed by Dyreson and Snodgrass [9]. Here, valid-time indeterminacy is coped with by associating a period of indeterminacy with a tuple. A period of indeterminacy is a period between two indeterminate instants, each one consisting of a range of chronons and of a probability distribution over it. Since the ranges of chronons defining the starting and ending points of a period cannot overlap, periods of indeterminacy must have at least one "determinate" chronon. Thus, indeterminacy about existence cannot be expressed, and, disregarding probabilities, Snodgrass and Dyreson's approach is strictly enclosed in our *D+I* representation as regards expressiveness.

In the line of Artificial Intelligence research, Brusoni et al. [3] and Koubarakis [17] independently proposed a different approach, addressing indeterminacy in the context of temporal constraints between tuples, with specific attention to relative times.

Finally, it is worth mentioning that, indeed, temporal indeterminacy is just a specific case of incomplete information. Many approaches have been developed to cope with incomplete information in relational databases (see, e.g., the extensive bibliography in Lipski [18] as regards early works, and Grahne [12]). For instance, Imielinski and Lipski [13] have identified precise conditions that should be satisfied by usual algebraic relational operators to meaningfully cope with relations where various kinds of "null values" are allowed. Gadia [11], e.g., provided a "bridge" between works on incomplete information and temporal relational databases. Interestingly, Gadia introduced *partial temporal elements* (and *set operators* on them) to cope with temporally indeterminate information, that closely resemble our DITEs (and set operators on DITEs). Also, Gadia et al. cope with values whose occurrence is uncertain, thus considering a form of what we term existential indeterminacy.

7 Conclusions

Though temporal indeterminacy is inherent in many real-world domains, it has received relatively limited attention within the database literature. In particular, the identification and analysis of different representations and of set operators for indeterminate temporal elements has not been adequately explored within the specialized literature. In this paper, we address this limitation. We identify a spectrum of approaches (data models, each with set operators and relational algebraic operators) to treat valid-time indeterminacy, and analyze their properties and their suitability to model phenomena in a compact way.

The incorporation of a refinement into a representation may improve not only the expressiveness of the representation, but also its compactness and suitability, thus possibly making it more "natural" to use. Therefore, among suitable representations, $D{+}I^{n,N,*}$ seems to be the best choice if one wants to reconcile the full data expressiveness of *RA* with compactness, since $D{+}I^{n,N,*}$ supports a determinate component, temporal dependence and cardinality constraints in a compact way. However, refinements have their own cost, especially in terms of the evaluation of set operations on temporal components. Therefore, we think that there is no "best representation" per se: the main contribution of Table 1 is to indicate users and developers the representation "*best suited*" to model the specific application they work with. For example, in case a user needs to compactly express determinate chronons, but she does not need to cope with either non-independent indeterminate chronons, or cardinality constraints, Table 1 shows that, for that particular situation, the *D+I* representation is the "best suited" one. This choice of the "best suited" representation might be done by the DB administrator, on the basis of the specific domain/application. Specifically, we envision the development of a user-friendly interface to help administrators in this choice (e.g., by exemplifying the choice criteria summarized in Table 1). Indeed, the "best suited" representation could also be chosen at a finer granularity, i.e., on a per-tuple basis. In such a context, additional operators must be provided to convert representations as needed based on their relative expressiveness. This is a goal of our future work.

Also, the lattice in Figure 2 can be used as a framework to analyze the expressiveness of current and (possibly) future representations in the literature.

As future work, we wish also to extend our approach to consider other refinements besides the ones described in Section 4. A practically relevant issue is *convexity* since in many domains indeterminacy at chronon *c* is correlated with indeterminacy near *c*. For instance, for *convex ITEs*, *Ext*(<{2,3,4}>) could be {∅, {2}, {3}, {4}, {2,3}, {3,4}, {2,3,4}} (thus excluding {2,4} which is not convex). However, the problem of providing a semantically correct definition of set operators for such representations requires further investigations. Additionally, a task of our future work is to extend also the other compact representations to cope with probabilities, applying the methodology of Section 5. Finally, our long-term goal is the development of suitable extensions to the SQL standard to treat the different forms of indeterminacy considered in this paper.

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