TEXTILE MATHEMATICS

PART II

BY

THOMAS WOODHOUSE

Head of the Weaving and Designing Department,
Dundee Technical College and School of Art

AND

ALEXANDER BRAND

Chief Draughtsman, Messrs. Douglas Fraser
& Sons, Ltd., Textile Engineers and
Ironfounders, Arbroath

BLACKIE AND SON LIMITED
50 OLD BAILEY LONDON
GLASGOW AND BOMBAY
1921

BY
THOMAS WOODHOUSE
AND
ALEXANDER BRAND

Textile Mathematics
In two parts

Textile Mechanics

Textile Machine Drawing

BLACKIE AND SON, LIMITED

PRINTED AND BOUND IN GREAT BRITAIN

CONTENTS

TEXTILE MATHEMATICS—II

CHAPTER I

RATIO, PROPORTION, AND VARIATION

Ratio.—In many textile calculations, and, indeed, in calculations generally, it is of importance to know the relation between two or more quantities. This relation is commonly expressed in the form of a fraction. For example, the relation between 1 yd. of cotton yarn and 1 hank (840 yd.) of cotton yarn is

$$\frac{1 \text{ yd.}}{840 \text{ yd.}} = \frac{1}{840}.$$

Again, the relation between 1 hank of cotton yarn and 1 yd. of cotton yarn is

$$\frac{840 \text{ yd.}}{1 \text{ yd.}} = \frac{840}{1}.$$

In each case the relative value of the numerator to the denominator is termed the ratio. The ratio between 1 yd. and 1 hank is $\frac{1}{840}$; while the ratio of 1 hank to 1 yard is $\frac{840}{1}$, or simply 840. The first result implies that 1 yd. is the $\frac{1}{840}$th part of 1 hank, and the second result implies that 1 hank is 840 times the length of 1 yd., i.e. that there are 840 yd. in 1 hank of cotton.

In general, the ratio of any quantity a to any other quantity b is expressed by the fraction $\frac{a}{b}$; a and b are called the terms of the ratio. When a is greater than b, the ratio is called a ratio of greater inequality; and when a is less than b, the ratio is one of less inequality; when a is equal to b, the ratio is unity or 1.

PROPORTION.—Let p, q, r and s be four quantities of such value that the ratio of p to q is equal to the ratio of r to s; that is to say

$$\frac{p}{q} = \frac{r}{s}.$$

When the relation between four quantities may be thus expressed, it is a common practice to state it in the following form:

$$p : q = r : s.$$

Read: the ratio of p to q equals the ratio of r to s.

An older method of showing the relation is as follows:—

$$p : q :: r : s.$$

Read: p is to q as r is to s.

The four quantities are said to be in proportion, and each is termed a proportional.

A deduction of great value in many types of calculations may be made from the expression

$$\frac{p}{q} = \frac{r}{s}.$$

Thus, if we multiply each side of the equation by q, we obtain

$$\frac{p \times q}{q} = \frac{r \times q}{s}.$$

Now multiply each side of the new equation by s, and we obtain:

$$\frac{p \times q \times s}{q} = \frac{r \times q \times s}{s}.$$

If like terms in the numerator and denominator of each side of the equation be cancelled, thus,

$$\frac{p \times q \times s}{q} = \frac{r \times q \times s}{s},$$

the result is

$$p \times s = r \times q.$$

If the latter equation be compared with the original one,

$$p : q = r : s,$$

it will be seen that the product of the first and last terms, i.e. p and s, is equal to the product of the second and third terms, q and r, and indicated below,

$$\underbrace{p : \overbrace{q = r} : s}.$$

This conclusion, when stated in more definite mathematical language, shows that when any four quantities, such as p, q, r and s are in proportion, the product of the "means" (or the two middle quantities) equals the product of the "extremes" (the two end quantities).

VARIATION.—In many calculations there are certain pairs of quantities which vary one with the other;

when one quantity is large, the other is also large, and when one is small, the other is small. But, however much the quantities themselves may vary, the ratio between them is always the same.

If t and v are two such quantities, and of such a value that the ratio $\frac{t}{v}$ is *always* the same, t is said to vary as v. This relation, for brevity, is usually written: $t \propto v$. It will be evident that for any other values of t and v, such as t_1 t_2 t_3, &c., and v_1, v_2, v_3 &c.,

$$\frac{t}{v} = \frac{t_1}{v_1} = \frac{t_2}{v_2} = \frac{t_3}{v_3} \ldots\ldots\ldots\ldots \frac{t_n}{v_n}.$$

In such a case as the above, where the ratio $\frac{t}{v}$ is constant, it is obvious that the larger t becomes the larger must v become also. It is equally obvious that, if t be made smaller, v must be made smaller in order to keep the ratio constant. All these conclusions may be summed up tersely, thus,

if t varies as v, then $\frac{t}{v} = \text{C}$ (a constant), and the two quantities vary "directly".

Sometimes it is found that one quantity gets smaller as the other quantity gets larger, and vice versa. In this case one quantity is said to vary "inversely" as the other quantity. Therefore, if t and v are again the letters representing two different quantities, t is said to vary as $\frac{1}{v}$, i.e. t varies as the reciprocal of v, and $t \times v = \text{C}$ (a constant).

Practical examples of the above principles will be found in the working out of the following examples.

Example 1.—A factory boiler, during a 10-hour trial, consumed 40 cwt. of coal. The ash, clinker, &c., drawn from the fire during the trial amounted to $1\frac{1}{2}$ cwt. What is the ratio of ash, &c., to coal? Express the ratio as a percentage.

$$\text{Ratio} = \frac{\text{Ash, \&c.,}}{\text{Coal}} = \frac{1\frac{1}{2}\text{ cwt.}}{40\text{ cwt.}} = \frac{3}{2 \times 40} = \frac{3}{80} = \frac{0{\cdot}0375}{1}.$$

Ratios are often expressed as percentages for convenience in comparison. The introduction of the percentage idea necessitates an existing ratio. In other words, a second ratio is to be found of which one term, 100, is known. Thus, in the above case,

$$\frac{3}{80} = \frac{x}{100},$$

where x is the percentage of ash, &c., to coal.

By arranging these as already shown we have:

$$3 : 80 = x : 100,$$

and since the product of the extremes equals the product of the means, we see that

$$80\,x = 3 \times 100,$$

or by cross multiplication as indicated below:

$$\frac{3}{80} \times \frac{x}{100},$$

$$\text{we get } 80\,x = 3 \times 100.$$

$$x = \frac{3 \times 100}{80},$$

$$= 3{\cdot}75 \text{ per cent ash, \&c.}$$

Once the above principle is understood, the intermediate steps can be omitted, and the ratio may be expressed as a percentage by simply multiplying the ratio by 100. Thus,

$$\frac{3}{80} \times 100 = \frac{300}{80} = 3{\cdot}75 \text{ per cent as before.}$$

Example 2.—A shaft running at 250 revolutions per minute carries a 19-inch diameter pulley. The latter drives by means of a belt a 12-inch diameter pulley on a slubbing or roving frame. Find the speed in r.p.m. of the frame pulley, and the speed of the spindles, if the ratio between spindle speed and pulley speed is 7 to 4.

Since the driving and driven pulleys are connected by a belt, the surface speeds of the two pulleys, neglecting slip, must be alike. If D is the diameter of the driving pulley, and n its speed in r.p.m., and d is the diameter of the driven pulley, and N its r.p.m., then,

$$\pi D n = \pi d N,$$
$$\text{whence } Dn = dN \text{ (after dividing each side by } \pi\text{)},$$
$$\text{and } N = \frac{Dn}{d} \text{ (after dividing each side by } d\text{)}.$$
$$\therefore N = \frac{19 \times 250}{12} = 395{\cdot}83 \text{ r.p.m.}$$

Comparing the two speeds and diameters, one finds,

Driving pulley, 19 in. diameter and 250 r.p.m.
Driven pulley, 12 in. diameter and 395·83 r.p.m.

In the case of the driving pulley, there is a large diameter and a low speed; in that of the driven pulley, a small diameter and a high speed. It is not difficult to see, and to state as a general rule

that, the diameters of pulleys connected by a belt are inversely proportional to the revolutions per minute of the pulleys.

It is further stated in Example 2 that the ratio of the spindle speed to the pulley speed is 7 to 4, i.e.

$$\frac{\text{Spindle speed}}{\text{Pulley speed}} = \frac{7}{4} = \frac{x}{395{\cdot}83}.$$

In other words, find a second ratio equal to $\frac{7}{4}$, and of which 395·83 is one term.

$$\frac{7}{4} = \frac{x}{395{\cdot}83},$$
$$4x = 7 \times 395{\cdot}83,$$
$$x = \frac{7 \times 395{\cdot}83}{4} = 692{\cdot}7 \text{ r.p.m.},$$

so that when the pulleys of the slubbing frame run at 395·83 r.p.m., the spindles run at 692·7 r.p.m.

Example 3.—A shaft runs at 180 r.p.m., and it is desired to drive a machine at 135 r.p.m.; if the pulley on the machine is 16 in. in diameter, what size of drum should be placed on the shaft?

$$\text{Drum} \times \text{r.p.m.} = \text{machine pulley} \times \text{r.p.m.}$$
$$180 \times D = 135 \times 16,$$
$$D = \frac{135 \times 16}{180}.$$
$$\therefore D = 12 \text{ in. diameter.}$$

Exercises, with answers, on p. 91.

CHAPTER II

AVERAGES

Many types of textile calculations depend for their solution on an exact knowledge of what is implied by the term average. The average of a series of numbers may be defined as the "mean" or "middle" number of the series. Suppose, for example, it is intended to produce cloth in a loom with 40 picks or shots per inch, and in the weaving process the regularity of the shotting is questioned. It is decided to test the number of shots by actual measurement of the cloth and the counting of the shots. It is found that—

In the first	inch there are	41	shots	
,, second	,, ,,	40	,,	
,, third	,, ,,	39	,,	
,, fourth	,, ,,	39	,,	
,, fifth	,, ,,	42	,,	
		201	,,	in 5 in.

$$\therefore \frac{201 \text{ shots}}{5 \text{ in.}} = 40{\cdot}2 \text{ shots per inch,}$$

which is the mean or average number of shots per inch in the 5 in. tested.

The average A of any series of quantities a, b, c, d, e, f, g, &c., may therefore be expressed as under:

$$A = \frac{a + b + c + d + e + f + g + \&c.}{N},$$

where N is the number of quantities in the numerator. The result may be expressed in words, thus,

$$\text{Average} = \frac{\text{Sum of the quantities}}{\text{Number of the quantities}};$$

or, symbolically:

$$A = \frac{S}{N}.$$

It is of importance to note that the above equation may also be written,

$$S = NA,$$

which indicates that the "sum" of any series of quantities is equal to the product of the number of quantities and the "average" quantity.

Example 4.—The following table gives, for the years indicated, the net imports of wool from Australia and New Zealand in millions of pounds. Find the yearly average of each, showing which is the greater, and by how much.

Year.	Australia.	New Zealand.
1907	144	134
1908	111	137
1909	105	132
1910	137	153
1911	158	140
5 years	655	696

$$\text{Australian average} = \frac{655}{5} = 131 \text{ million lb.}$$

$$\text{New Zealand average} = \frac{696}{5} = 139{\cdot}2 \text{ million lb.}$$

$139{\cdot}2 - 131 = 8{\cdot}2$ million lb. is the average yearly difference in favour of New Zealand.

It will now be understood that if there are x quantities of a, y quantities of b, z quantities of c, &c., their average will be,

$$A = \frac{xa + yb + zc}{x + y + z}.$$

This is merely another form of the fundamental equation given above, extended to take in a series of numbers of quantities, instead of only a simple series of quantities.

Example 5.—A cotton mill contains 4 ring frames of 480 spindles each, 20 frames of 456 spindles each, and 18 frames of 432 spindles each, all of different gauges to suit the range of counts required to be spun. Find the average number of spindles per frame.

$$A = \frac{xa + yb + zc}{x + y + z}$$

$$= \frac{(4 \times 480) + (20 \times 456) + (18 \times 432)}{4 + 20 + 18}$$

$$= \frac{1920 + 9120 + 7776}{42}$$

$$= \frac{18816}{42} = 448, \text{ average number of spindles per frame.}$$

The average of a series of quantities is sometimes also called the "arithmetic mean". The word "mean" is often used in this sense in place of average. Thus, in the case of an indicator diagram, the "mean effective pressure" is required in order to be able to calculate the horse-power developed. The mean effective pressure is really represented by the mean or average height of the ordinates of the diagram, an ordinate being a straight line bounded by the diagram and perpendicular to some standard line, usually taken as horizontal, or in the direction of the greatest length. Thus, in fig. 1, AB and CD are two ordinates of the indicator diagram reproduced, EF being the datum line to which they, and all the other ordinates, are perpendicularly drawn.

The average height of the whole diagram may be

found by dividing the length EF into 10 strips, all equal in width, finding the average height of all the strips by measurement, as indicated by the dotted ordinates, and then obtaining the mean height of the diagram by finding the average of the 10 measurements.

Example 6.—In an engine indicator diagram such as that at ABDC in fig. 1, the heights at the middle line of each strip are respectively ·54, ·66, ·74, ·87,

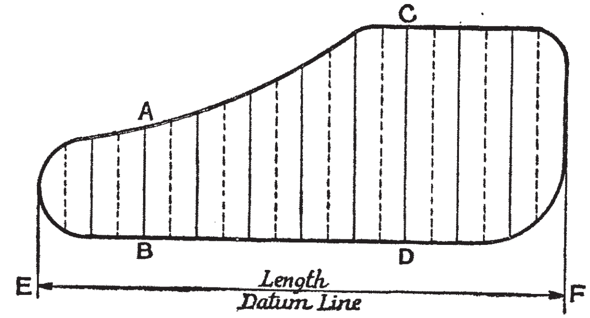


Fig. 1

1·02, 1·20, 1·35, 1·35, 1·35, and 1·19 in. Find the mean height, and the mean effective pressure, if the scale be $\frac{1}{60}$, *i.e.* $\frac{1}{60}$th of an inch equals 1 lb. per square inch of pressure.

Average height

$$= \frac{\cdot54 + \cdot66 + \cdot74 + \cdot87 + 1\cdot02 + 1\cdot20 + 1\cdot35 + 1\cdot35 + 1\cdot35 + 1\cdot19}{10}$$

$$= \frac{10\cdot27}{10} = 1\cdot027 \text{ in.}$$

Mean pressure = 1·027 × 60 lb. per square inch.
= 61·62 lb. per square inch.

It was shown in Part I, Chap. V, that the area of a rectangle is equal to the product of the length and height. This formula or rule, although very simple in its terms, is extremely important, as by a slight extension in the meaning of one of its terms it may be used to find the area of any closed figure.

Let ABCDE, fig. 2, represent any closed figure. Its mean height, measured from the base AB, may be found in a manner similar to that used for the indicator diagram in Example 6. Let the mean height thus found be BX. It is evident that a rectangle may

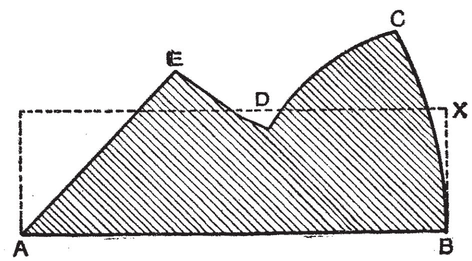


Fig. 2

be constructed with length AB and height BX, the area of which is equal to the area of the irregular closed figure.

Area of closed figure = area of rectangle.
= length × breadth or height.
= AB × BX.
= length × mean height.

This may be verified in a simple manner by referring to Part I, Chap. VII, where the method of finding the area of a trapezium is discussed. There it is shown that:

Area of trapezium = half the sum of the parallel sides × perpendicular distance between them.

Referring to fig. 3, ABCD is a trapezium, with

sides AB and CD parallel, and the distance between them = BC. BC may, however, be regarded as the length of the diagram. The height, measured from BC, varies at every point from BA to CD. CD is greater than the mean height by just as much as BA

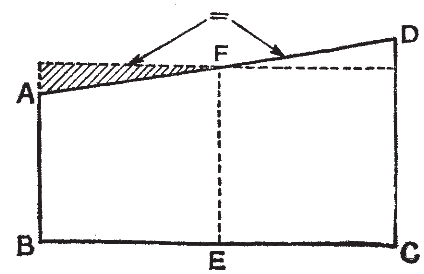


Fig. 3

is less than the mean height (see shaded and unshaded triangles). It is evident that the mean height EF is the average of BA and CD $= \frac{BA + CD}{2} =$ half the sum of the parallel sides.

Consequently, in place of the rule quoted above, one may use the one just deduced, i.e.:

Area of trapezium = length × mean height.

Exercises, with answers, on p. 93.

CHAPTER III

PERCENTAGES

In the comparison of quantities, it often becomes necessary to express one quantity as a part or fraction of another. This may be done by expressing the ratio or proportion of the two quantities as a fraction. For example, if 500 lb. of grey or natural-coloured yarn are sent to the bleachfield, the bleaching processes remove some of the weight; suppose that 50 lb. weight is thus lost, and 450 lb. of yarn are returned to the spinner. The actual loss in weight is

$$500 \text{ lb.} - 450 \text{ lb.} = 50 \text{ lb.},$$

and the proportion of lost material being 50 lb. out of 500 lb., or 1 lb. out of every 10, the ratio of the loss to the original weight is

$$\frac{50 \text{ lb. loss}}{500 \text{ lb. original weight}} = \frac{50}{500} = \frac{1}{10}$$

If the denominator of this fraction is altered by any means to 100, the ratio is expressed as a "percentage", that is to say, it is expressed in hundredths. This method of expressing ratios or proportions is very convenient, and is much used for purposes of comparison.

The loss on the yarn in the above case is seen to be $\frac{1}{10}$ of the original weight of the yarn. Now, $\frac{1}{10} = \frac{10}{100}$; the loss may therefore be stated as 10 per cent, literally meaning 10 per centum, or 10 per hundred, and implying that the loss due to bleaching amounted to 10 lb. out of every 100 lb. treated.

The following examples illustrate different classes of percentage problems, and will enable the student to solve successfully other problems of a similar nature.

Example 7.—In 1912, the total textile exports from the United Kingdom amounted to £188,400,000, made up as in the table below, where the figures represent millions of pounds:

Cotton	122·2
Wool	37·8
Silk...	2·2
Jute	3·6
Linen	9·7
Clothing, &c. ...	12·9
Total ...	188·4

Arrange these exports in the order of importance with regard to export value, expressing each value as a percentage of the whole.

NOTE.—Percentage is usually expressed by the sign %. 2½ % is read: two and a half per cent.

$$\text{Cotton: } \frac{\text{Cotton value}}{\text{Total value}} \times 100 = \frac{122{\cdot}2 \times 100}{188{\cdot}4} = 64{\cdot}86\,\%$$

$$\text{Wool: } \frac{\text{Wool value}}{\text{Total value}} \times 100 = \frac{37{\cdot}8 \times 100}{188{\cdot}4} = 20{\cdot}06\,\%$$

$$\text{Silk: } \frac{\text{Silk value}}{\text{Total value}} \times 100 = \frac{2{\cdot}2 \times 100}{188{\cdot}4} = 1{\cdot}17\,\%$$

$$\text{Jute: } \frac{\text{Jute value}}{\text{Total value}} \times 100 = \frac{3{\cdot}6 \times 100}{188{\cdot}4} = 1{\cdot}91\,\%$$

$$\text{Linen: } \frac{\text{Linen value}}{\text{Total value}} \times 100 = \frac{9{\cdot}7 \times 100}{188{\cdot}4} = 5{\cdot}15\,\%$$

$$\text{Clothing, \&c.: } \frac{\text{Clothing, \&c., value}}{\text{Total value}} \times 100 = \frac{12{\cdot}9 \times 100}{188{\cdot}4} = 6{\cdot}85\,\%$$

Total, 100·00 %

These are arranged in order of importance with regard to export value: 1, Cotton 64·86%; 2, wool, 20·06%; 3, clothing, &c., 6·85%; 4, linen, 5·15%; 5, jute, 1·91%; 6, silk, 1·17%.

The principle just enunciated can be expressed algebraically as follows:

Let p = a part or proportion of any quantity.
Q = the quantity.
r = the rate per cent. Then:

$$\frac{p}{Q} \times 100 = r.$$

This may, of course, be arranged to read:

$$r = \frac{100\,p}{Q}$$

It is wise to notice that in all there are 4 quantities involved, of which all are variable except the 100. The value of r is shown above; the student will easily find the values for p and Q.

Example 8.—A dressing mixture recommended for cotton damasks is made up of 68% water, 10% wheat starch, 7% tragasol, 14% China clay, and 1% glycerine. Find the proportions of each to make 1500 lb. weight of mixture.

Water: 68% of 1500 lb. $= \frac{68 \times 1500}{100} = 1020$ lb.

which at 10 lb. per gallon = 102 gal.

Wheat starch: 10% of 1500 lb. $= \frac{10 \times 1500}{100} = 150$ lb.

Tragasol: 7% of 1500 lb. $= \frac{7 \times 1500}{100} = 105$ lb.

China clay: 14% of 1500 lb. $= \frac{14 \times 1500}{100} = 210$ lb.

Glycerine: 1% of 1500 lb. $= \frac{1 \times 1500}{100} = 15$ lb.

100% — 1500 lb.

Example 9.—A beam of bleached and dressed (starched) linen warp contains 116 lb. weight of yarn. In the bleaching process the yarn lost 12% of its original weight, and in the dressing gained 5% of its bleached weight. Find the original weight of the yarn.

Let 100% = the original weight.
12% = the loss in bleaching.
88% = the bleached weight.

Now, the yarn gains 5% of the bleached weight in dressing, i.e.:

$\frac{5}{100}$ of 88% = 4·4%.
∴ Dressed weight = 88% + 4·4%.
= 92·4% of the original weight,

so that 92·4% of the original weight of the yarn is 116 lb.; therefore, the original weight was

$$\frac{116 \times 100}{92 \cdot 4} = 125 \cdot 54 \text{ lb.}$$

The result may be worked out shortly as follows:

$$\text{Actual weight} \times \frac{\text{undressed weight}}{\text{dressed weight}} \times \frac{100}{88} = \text{original weight.}$$

$$116 \times \frac{100}{105} \times \frac{100}{88} = \frac{29000}{231} = 125 \cdot 54 \text{ lb.}$$

The result may also be checked in this manner:

Original weight = 125·54 lb.
12% loss in bleaching = $\frac{12}{100}$ of 125·54
= 15·05 lb.
∴ Bleached weight = 125·54 − 15·05
= 110·49 lb.
5% gain in dressing = $\frac{5}{100}$ of 110·49
= 5·52.
∴ Actual weight = 110·49 + 5·52
= 116·01 lb.

The slight discrepancy is due solely to the fact that all decimal figures beyond the second have been neglected.

Example 10.—$\frac{1200 \times 420}{12 \times 840} = 50$ lb. shows the method of finding the weight in pounds of a warp 420 yd. long containing 1200 threads of 12^s grey or natural cotton. If the original grey yarn were correct in count, and if in the bleaching process 5 % of the weight were lost, what percentage of starch or size should be added in the slashing process to obtain the original weight of 50 lb.?

$$\frac{1200 \times 420}{12 \times 840} \times \frac{95}{100} = 47 \cdot 5 \text{ lb. in bleached state,}$$

$$\text{then } 47 \cdot 5 \text{ lb.} + \frac{x\%}{100} \times 47 \cdot 5 = 50 \text{ lb.}$$

$$\frac{47 \cdot 5x}{100} = 50 \text{ lb.} - 47 \cdot 5 \text{ lb.}$$

$$x = \frac{2 \cdot 5 \times 100}{47 \cdot 5}.$$

$$\therefore \text{ Amount of starch } = 5\tfrac{5}{19}\%.$$

Or by the direct method as in Example 9.

$$\frac{1200 \times 420}{12 \times 840} \times \frac{95}{100} \times \frac{100 + x}{100} = 50 \text{ lb.}$$

The student should work this out to find the value of x, the percentage amount of starch to be added.

Exercises, with answers, on p. 94.

CHAPTER IV

LOSS AND REGAIN

Practically all textile fibres are capable of absorbing a considerable amount of moisture without appearing to be actually damp or wet. Wool, for example, after it has been dried in air, may contain 8 to 14 per cent of this hygroscopic moisture. If the air itself is saturated with moisture, and the wool exposed to it, the percentage of moisture may rise to 30 per cent.

Since most, and in reality all, of the fibres are bought and sold by weight, it is evident that the actual amount of fibre, and the actual amount of moisture absorbed by the fibre, are matters of great importance. A parcel of wool may weigh 100 lb., but when the water has been evaporated it may weigh only 70 lb. The absolute dry condition of the fibre, however, is not natural; but it is desirable, and indeed essential, that the fibre should not contain too much moisture; as a matter of fact, a fixed quantity of moisture should be allowed, or otherwise trading would become difficult.

So important is this question that laboratories, called "conditioning houses", have been established in the various large textile centres, and equipped with apparatus for determining the amount of moisture in textile materials of all kinds. Thc apparatus consists of an oven, a wire cage to hold the fibre, and scales and weights to register the exact weight of the material when it entered, and after it has been thoroughly dried.

After the absolute dry weight of the fibre has been obtained in this way, the normal weight is calculated by adding to the dry weight the amount of moisture supposed to be present in the material under natural conditions, i.e. normal conditions of temperature and humidity. This added amount is termed regain. The percentage of regain allowable varies in different centres according to local conditions, although attempts have been made to fix the amounts by international agreement. The percentage of regain is different for the various kinds of fibres, while it also differs according to the state of the fibre, i.e. whether it is in the form of yarn, cloth, &c. For example, the Bradford Conditioning House has established the following allowable regains for the various fibres and the different stages of manufacture:—

Wools and waste	... ...	16 per cent.
Tops combed with oil	... ...	19 ,,
,, ,, without oil	...	$18\frac{1}{4}$,,
Noils, ordinary	... ...	14 ,,
Noils, clean	... ...	16 ,,
Worsted yarns	... ...	$18\frac{1}{4}$,,
Cotton yarns	... ...	$8\frac{1}{2}$,,
Silk yarns	... ...	11 ,,
Worsted and woollen cloths	...	16 ,,

The International Congress at Turin fixed the following as the allowable percentages of regain for the fibres named:—

Silk	... ...	11 per cent.
Wool, tops	... ...	$18\frac{1}{4}$,,
,, yarns	... ...	17 ,,
Cotton	... ...	$8\frac{1}{2}$,,
Linen	... ...	12 ,,
Hemp	... ...	12 ,,
Jute	... ...	$13\frac{3}{4}$,,
Phormium fibre (New Zealand)		$13\frac{3}{4}$,,

In the conditioning of any textile material there are numerous types of calculations involved, the mathematical principles of which will be discussed immediately. The two chief types involve:

(*a*) The calculation of the amount of moisture present in the sample tested and expressed as a percentage of the original weight; and

(*b*) The determination of the conditioned weight of the material, allowing a definite percentage of regain, this percentage being based upon the dry weight of the material.

Example 11.—A quantity of material weighing w lb. is tested for moisture, and after drying is found to weigh d lb. The loss of weight in drying, which is equal to the amount of moisture in the sample, is:

wet weight − dry weight,

or $w - d$.

$\therefore \frac{w-d}{w}$ = the proportion of moisture in respect of the original weight,

hence, $\frac{w-d}{w} \times 100 = p$, the percentage of moisture in the sample.

Example 12.—Suppose a 1 lb. sample of cotton from a delivered bale is found to weigh 14 oz. after drying, find the percentage of moisture.

16 oz. − 14 oz. = 2 oz. moisture,

$\frac{2 \text{ oz.}}{16 \text{ oz.}} \times 100 = 12\frac{1}{2}$ per cent moisture.

It is shown above that

$$p = \frac{w-d}{w} \times 100,$$

from which it is possible to deduce a rule for finding the weight of the dry material d, given the original weight and the percentage loss.

$$p = \frac{(w-d)\,100}{w},$$

$$pw = (w-d)\,100,$$

$$pw = 100w - 100d,$$

$$100d = 100w - pw.$$

$$\therefore\ 100d = w(100-p),$$

$$\text{and } d = \frac{w(100-p)}{100}.$$

Example 13.—By mutual agreement $12\frac{1}{2}$ per cent moisture is allowed on a certain parcel of fibre; what should be the dry weight, after exposure in the oven, of 16 oz. of the original material?

$$d = \frac{w(100 - p)}{100}$$
$$= \frac{16(100 - 12\frac{1}{2})}{100} = \frac{16 \times 87\frac{1}{2}}{100}$$
$$= \frac{16 \times 175}{100 \times 2} = 14 \text{ oz. dry fibre.}$$

It is also possible to deduce an expression giving the original weight, if the dry weight and the loss per cent be known. Thus, starting again from the fundamental equation,

$$p = \frac{100(w - d)}{w},$$

we have
$$pw = 100w - 100d,$$
$$pw - 100w = -100d,$$
$$w(p - 100) = -100d,$$
$$w = \frac{-100d}{p - 100}.$$

Multiply numerator and denominator of the right-hand expression by -1, and we have,

$$w = \frac{100d}{100 - p}.$$

Example 14.—The dry weight d, and the percentage loss p, of a sample of fibre are 14 oz. and $12\frac{1}{2}$ per cent respectively, what was the original weight of the sample?

$$w = \frac{100 \times 14}{100 - 12\frac{1}{2}}$$
$$= \frac{1400}{87\frac{1}{2}} = \frac{1400 \times 2}{175}$$
$$= 16 \text{ oz.}$$

The second important type of conditioning problem involves the question of regain; it is most important to bear in mind that the percentage of regain is calculated on the dry weight.

Let c = the conditioned weight,
r = the percentage regain,
d = the dry weight of the material.

Conditioned weight = dry weight + definite percentage regain;

$$i.e., \; c = d + (r \text{ per cent of } d)$$
$$= d + \left(\frac{r}{100} \times d\right)$$
$$= d + \frac{rd}{100}$$
$$= d\left(1 + \frac{r}{100}\right).$$

Example 15.—A regain of 19 per cent is allowed on a parcel of wool; if the dry weight is 14 oz., find the conditioned weight.

$$c = d\left(1 + \frac{r}{100}\right)$$
$$= 14\left(1 + \frac{19}{100}\right)$$
$$= 14\left(\frac{100}{100} + \frac{19}{100}\right)$$
$$= \frac{14 \times 119}{100}$$
$$= 16{\cdot}66 \text{ oz.}$$

If it were desired to find d or r when the other terms are known, we should have:

$$c = d\left(1 + \frac{r}{100}\right);$$

$$\text{hence} \quad d = \frac{c}{1 + \frac{r}{100}},$$

which, after simplification, becomes:

$$d = \frac{100c}{100 + r}.$$

We might test the latter by introducing the values from Example 15.

$$d = \frac{100c}{100 + r}$$

$$= \frac{100 \times 16{\cdot}66}{100 + 19} = 14 \text{ oz.}$$

The student should find r, having been given c and d.

Exercises, with answers, on p. 96.

CHAPTER V

MIXTURES: PROPORTIONS AND COSTS

Problems involving averages and percentage find a very practical application in questions regarding mixtures of all kinds. A slight study of the economics of production in the textile industry reveals the fact that spinners and manufacturers are com-

pelled for competitive reasons to produce certain grades of yarn and cloth at very low prices. These low prices prohibit the use exclusively of good-class material, and, as a general rule, low-class material cannot be profitably manufactured alone. By judicious mixing of the high and low qualities, however, it is possible to obtain yarns and cloth which can be sold at low prices, and yet at the same time give comparatively little trouble in the various processes of manufacture.

The typical problems involved are two in number:

1. To find the average cost per lb. of a mixture, given the proportions of the various materials and the cost of each.
2. To find the proportions of the various materials in a mixture when the cost per lb. of each constituent of the mixture and of the mixture itself are known.

The mathematical principle underlying the first type has already been discussed (see Example 5, p. 10). However, it may be useful to see the principle applied to the particular kind of problem indicated above.

To make the problem as clear as possible, we shall take first of all a numerical one.

Example 16.—Suppose that a mixture yarn is to be spun from a blend consisting of 45 per cent wool at 3*s*. 9*d*. per lb., 35 per cent cotton at 1*s*. 3*d*. per lb., and 20 per cent waste at 4*d*. per lb. Then:

									£	s.	d.
Wool	=	45 %	=	45 lb.	out of 100 lb. at 3*s*. 9*d*.	per lb.	=		8	8	9
Cotton	=	35 %	=	35 lb.	,, 100 lb. at 1*s*. 3*d*.	,,	=		2	3	9
Waste	=	20 %	=	20 lb.	,, 100 lb. at 0*s*. 4*d*.	,,	=		0	6	8
Mixture	=	100 %	=	100 lb.				which cost	10	19	2

$$\text{Average cost per lb.} = \frac{\text{Total cost}}{\text{Total weight in lb.}}$$

$$= \frac{£10,\ 19s.\ 2d.}{100 \text{ lb.}} = \frac{2630d.}{100 \text{ lb.}}$$

$$= 26{\cdot}3d. \text{ per lb.}$$

The general case may now be stated: If a lb. of material at A pence per lb., b lb. of material at B pence per lb., c lb. of material at C pence per lb., &c., are blended, the cost P of the mixture in pence per lb. will be:

$$P = \frac{aA + bB + cC + \&c.}{a + b + c + \&c.}$$

If reference be made to p. 10, it will at once be seen that this particular type of problem is but a variation of the example discussed there.

The second type of problem is of a more difficult nature. Keeping to the same notation as used in the last example, and referring to type (2) above, it is seen that P as well as A, B, C, &c., are known, and that it is required to find the quantities a, b, c, &c.

A simple example will suffice for demonstration.

Example 17.—Suppose it is desired to mix a lb. of material at A pence per lb. with b lb. of material at B pence per lb., in order to produce $(a + b)$ = say c lb. of a mixture at C pence per lb. Then aA pence and bB pence are the individual costs, while cC is the total cost; that is to say,

a lb. at A pence per lb.	=	aA pence
b lb. at B ,, ,,	=	bB pence
$(a + b)$ lb. of mixture	=	$(aA + bB)$ pence
or c lb. ,,	=	cC pence.

We might present the problem in another way, thus:

$$\begin{aligned} aA + bB &= cC, \\ \text{and} \quad (a + b)C &= cC, \text{ since } a + b = c, \\ \text{whence} \quad aA + bB &= (a + b)C, \\ aA + bB &= aC + bC; \\ \text{or} \quad Aa + Bb &= Ca + Cb, \\ Aa - Ca &= Cb - Bb, \\ a(A - C) &= b(C - B), \\ \frac{a(A - C)}{b} &= C - B, \\ \frac{a}{b} &= \frac{C - B}{A - C}; \end{aligned}$$

or, if desired, $a : b = C - B : A - C$.

$\frac{a}{b}$ = the ratio of the weights of the constituents of the mixture.

$C - B$ = the difference between the cost per lb. of the mixture and the cost per lb. of one of the constituents, b.

$A - C$ = the difference between the cost per lb. of one of the constituents, a, and the cost per lb. of the mixture.

The result that $\frac{a}{b} = \frac{C - B}{A - C}$ proves that the ratio of the weights of the constituents is equal to the ratio of the differences in costs per lb. of each constituent and the price per lb. of the mixture.

Example 18.—A flax tow yarn is to be made from Irish tow at 1*s.* or 12*d.* per lb., and Dutch tow at 10*d.* per lb. Find the proportion of each if the mixture is to be worth $10\frac{1}{2}d.$ per lb.

Using the same notation as in Example 17, we have,

$$\frac{a}{b} = \frac{C - B}{A - C}, \quad \text{when} \quad \begin{aligned} A &= 12d. \\ B &= 10d. \\ C &= 10\tfrac{1}{2}d. \end{aligned}$$

$$\frac{a}{b} = \frac{10\frac{1}{2} - 10}{12 - 10\frac{1}{2}}$$

$$= \frac{\frac{1}{2}}{1\frac{1}{2}}, \text{ or } \frac{1 \times 2}{2 \times 3}.$$

$$\therefore \frac{a}{b} = \frac{1}{3},$$

$$\text{or } \frac{1 \text{ unit at A, or } 12d. \text{ per lb.}}{3 \text{ units at B, or } 10d. \text{ per lb.}}$$

The result shows that if 1 lb. of Irish tow at 1*s.* per lb. is mixed with 3 lb. of Dutch tow at 10*d.* per lb., the mixture is worth 10½*d.* per lb. The result may be proved by working out the average cost per lb., using the quantities thus found.

1 lb. Irish	tow at 12*d.* per lb.	=	12*d.*
3 lb. Dutch	,, at 10*d.* ,,	=	30*d.*
4 lb. of mixture cost			42*d.*

$$\text{Average cost per lb.} = \frac{42d.}{4 \text{ lb.}} = 10\tfrac{1}{2}d.$$

The student is advised to consider the above method of finding the result in the form of a ratio of two originally unknown quantities, but another and much quicker way with two materials is to include only one unknown quantity as follows:—

Example 19.—Say 100 lb. of material is to be made from the above fibres at 12*d.* and 10*d.* respectively to produce a mixture at 10½*d.*

Let x = the no. of lb. of Irish tow at 12*d.*,

then 100 lb. $- x$ = the no. of lb. of Dutch tow at 10*d.*

$$12x + 10(100 - x) = 10\tfrac{1}{2} \times 100,$$

$$12x + 1000 - 10x = 1050,$$

$$2x = 1050 - 1000.$$

$$\therefore x = 25.$$

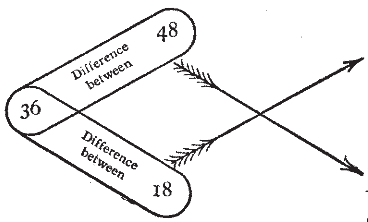
x =	25 lb. of Irish tow	at 12*d.* =	300*d.*
$(100 - x)$ =	75 lb. of Dutch ,,	at 10*d.* =	750*d.*
	100 lb. of mixture		1050*d.*

$$\therefore 1 \text{ lb.} = \frac{1050}{100} = 10\tfrac{1}{2}d.$$

In many cases, particularly those involving the mixture of more than two kinds of material, the mathematical relation becomes somewhat involved. The actual mathematical principles—except in practice—are often neglected in technical works and the "Alligation" method substituted. This method, however, is, in many cases, impracticable, and, in addition, it is not complete. For these reasons, only a simple example is shown, and it is not intended to present more elaborate examples in this work.

Example 20.—It is desired to make a mixture yarn from cotton at 1*s.* 6*d.* per lb. and wool at 4*s.* per lb., the resulting blend to be worth 3*s.* per lb. Find the relative proportions of each.

To avoid the use of fractions, change the prices to pence per lb., i.e. 18, 48, and 36. Place the price of the mixture on the left, and the two constituent prices on the right, as under:



18 lb. at 4*s*. = 72*s*.

12 lb. at 1*s*. 6*d*. = 18*s*.

30 lb. cost 90*s*.

And $\frac{90s.}{30\text{ lb.}} = 3s.$ per lb.

In each case the lesser value is subtracted from the greater, and the result placed in the position shown. Thus, difference between 36 and 48 is 12, i.e. 12 lb. of the lower-priced material. Difference between 36 and 18 is 18, i.e. 18 lb. of the higher-priced material.

Exercises, with answers, on p. 96.

CHAPTER VI

INDICES—USE OF LOGARITHMS

INDICES.—Use has already been made of such expressions as x^2, r^3, &c., and it will be useful to inquire more deeply into the meanings of the figures, 2, 3, &c., each of which, when placed in a plane a little higher than, and to the right of, a letter as exemplified, or in a similar position with regard to a number instead of a letter, is termed an "index" or "power".

If any quantity x be multiplied by itself, the result

is $x \times x$, or xx, or shortly x^2; the latter is read as x squared, or x to the power 2.

If any quantity r be multiplied by itself, and again multiplied by r, the result is $r \times r \times r = rrr = r^3$; the latter is read r cubed, or r to the 3rd power.

The figures 2 and 3 thus used indicate the "power" to which the quantity x or the quantity r has been raised; x^2 may thus mean x raised to the 2nd power, r^3 means r raised to the 3rd power, and generally, p^n means that a quantity p has been raised to the nth power: 2, 3, and n are the "indices" of the respective powers.

If x^2 is to be multiplied by x^3, the process may be illustrated as follows:

$$\begin{aligned} \text{The product } x^2 \times x^3 &= xx \times xxx \\ &= xxxxx \\ &= x^5. \end{aligned}$$

$$\begin{aligned} \text{Similarly, } p^4 \times p^5 &= pppp \times ppppp \\ &= ppppppppp \\ &= p^9. \end{aligned}$$

It is evident from the above two examples that the required products may be obtained by adding, in each case, the two indices. Thus:

$$x^2 \times x^3 = x^{2+3} = x^5$$

$$\text{and } p^4 \times p^5 = p^{4+5} = p^9.$$

In general, $x^m \times x^n = x^{m+n}$ where m and n are any numbers or quantities, whole, mixed, or fractional.

Again, if x^3 is to be divided by x^2, the process is equivalent to the following:

$$\frac{x^3}{x^2} = \frac{xxx}{xx} = x,$$

$$\text{and } p^5 \text{ divided by } p^3 = \frac{p^5}{p^3} = \frac{ppppp}{ppp} = pp = p^2.$$

It is again evident that the same result, in each case, could be obtained by subtracting the index of the term in the denominator from the index of the term in the numerator. Thus:

$$\frac{x^3}{x^2} = x^3 \div x^2 = x^{3-2} = x^1 = x,$$

$$\text{and } \frac{p^5}{p^3} = p^5 \div p^3 = p^{5-3} = p^2.$$

In general, $\frac{x^m}{x^n} = x^m \div x^n = x^{m-n}$ where m and n are any numbers or quantities.

One of the most important points to be noticed at present is the fact that by using the indices of quantities, the process of multiplication is replaced by that of addition, while the process of division is replaced by that of subtraction.

In general, where the numbers m and n are positive whole numbers, little trouble is experienced in working with indices, but when the index is a negative quantity, a fraction, a mixed number, or zero, some little difficulty is found in attaching a definite meaning to the expression. In this respect, the following examples are worth careful study:—

Example 21.—What is the meaning attached to the expression x^0, i.e. x raised to the power zero?

It is shown above that

$$x^m \times x^n = x^{m+n};$$

suppose m is equal to 0, then:

$$x^0 \times X^n = x^{0+n} = X^n.$$

Note the terms in large type; these show that if x^0 is multiplied by x^n, the result is x^n, i.e. x^0 is of such a value that when it is multiplied by x^n, the product is also x^n. x^0 must therefore equal 1. It is important to remember that x represents *any* number, so that any number, large or small, raised to the power zero is equal to 1.

Example 22.—What meaning is attached to the expression $x^{\frac{1}{2}}$?

As previously shown, $x^m \times x^n = x^{m+n}$.

Suppose m and n are each equal to $\frac{1}{2}$, then:

$$\begin{aligned} x^{\frac{1}{2}} \times x^{\frac{1}{2}} &= x^{\frac{1}{2}+\frac{1}{2}} \\ &= x^1 \\ &= x. \end{aligned}$$

$x^{\frac{1}{2}}$ is therefore that quantity which, multiplied by itself, is equal to x. It will thus be seen that $x^{\frac{1}{2}}$ is equal to the square root of x, since $\sqrt{x} \times \sqrt{x} = x$. It may be considered that in the power $\frac{1}{2}$, the denominator indicates the root of the quantity x, while the numerator indicates the power of the same quantity. Thus,

$$x^{\frac{1}{2}} = \sqrt[2]{x^1} = \sqrt{x}.$$

$$\text{Hence, } x^{\frac{1}{2}} = \sqrt{x}.$$

In the same way $x^{\frac{3}{4}}$ = the 4th root of x raised to the 3rd power,

$$\therefore x^{\frac{3}{4}} = \sqrt[4]{x^3}.$$

Example 23.—What meaning is attached to the expression x^{-1}, i.e. x raised to the power "minus 1"?

Again, in the product $x^m \times x^n = x^{m+n}$ let $m = 1$ and $n = -1$. Then:

$$x^m \times x^n \text{ becomes } x^1 \times x^{-1} = x^{1-1};$$
$$\therefore x \times x^{-1} = x^0$$
$$x \times x^{-1} = 1.$$

x^{-1} is therefore that quantity which, when multiplied by x is equal to 1; in other words x^{-1} and x are related quantities, such as $\frac{2}{3}$ and $\frac{3}{2}$, 4 and $\frac{1}{4}$, and the like, that is to say, they are reciprocal numbers. So that

$$x^{-1} = \frac{1}{x}.$$

In the same way:

$$x^{-3} = \frac{1}{x^3},$$
$$x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}},$$
$$\text{and } x^{-\frac{3}{4}} = \frac{1}{x^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{x^3}}.$$

LOGARITHMS.—Any number or quantity can be expressed as a power of 10. For example:

$$1 = 10^0$$
$$10 = 10^1$$
$$100 = 10^2$$
$$1000 = 10^3$$
$$3 \cdot 162 = 10^{\frac{1}{2}}, \text{ i.e. } \sqrt{10}$$
$$\cdot 1 = 10^{-1}, \text{ i.e. } \tfrac{1}{10}.$$

&c., &c.

These examples should be compared with the examples investigated with reference to the meaning of the indices; and, although the above examples have very evident relations with the base 10, in many cases the relation is by no means obvious. For instance, it can be proved that

$$31620 = 10^{4 \cdot 5},$$
$$\text{and that } 4 \cdot 216 = 10^{\cdot 6249}.$$

Let us suppose that 31620 is to be multiplied by $4 \cdot 216$, then, by the foregoing examples we should have:

$$31620 \times 4 \cdot 216 = 10^{4 \cdot 5} + 10^{\cdot 6249}$$
$$= 10^{4 \cdot 5 + \cdot 6249}.$$
$$= 10^{5 \cdot 1249}.$$

$10^{5 \cdot 1249}$ can be proved $\doteqdot$ 133,300, while, by actual multiplication, $31620 \times 4 \cdot 216 = 133{,}309 \cdot 92$. The slight discrepancy is due to the fact that the powers of 10 given are correct only to 4 significant figures. Much nearer results could be obtained by the use of what are termed 7-figure logarithms, but these will not be used in this work.

The above powers, $4 \cdot 5$, $\cdot 6249$, and $5 \cdot 1249$, and all such powers of 10, are called COMMON LOGARITHMS. If $31620 = 10^{4 \cdot 5}$, $4 \cdot 5$ is the logarithm of 31620 to the base 10. Similarly, if $4 \cdot 216 = 10^{\cdot 6249}$, $\cdot 6249$ is the logarithm of $4 \cdot 216$ to the base 10. In general, if any quantity N can be expressed as a power of 10, i.e.:

$$\text{If } N = 10^x,$$
$$\text{then } x = \log N \text{ to the base } 10$$
$$= \log_{10} N.$$

The abbreviation "log" is written for logarithm.

TABLES OF LOGARITHMS.—Tables of four-figure logarithms, i.e. logarithms correct to four significant figures, are given on pp. 112 to 115. The tables contain the common logarithms of all four-figure numbers between 1·0 and 9·999, arranged for convenience (1) in looking up the logarithms of given numbers (logarithms), and (2) for looking up the numbers corresponding to given logarithms (antilogarithms).

Strictly speaking there should be a decimal point in the tables after the first figure of the numbers (N) and before the first figure of the logarithms ($\log_{10}$ N); these decimal points are invariably omitted to simplify the printing of the tables. Keeping in view the fact that a common logarithm is the power to which ten must be raised to equal the given number, it is important to be able to write any number N in the form of $n \times 10^x$. For example:

$$\begin{aligned} 1728 &= 1{\cdot}728 \times 10^3, \\ 172{\cdot}8 &= 1{\cdot}728 \times 10^2, \\ 17{\cdot}28 &= 1{\cdot}728 \times 10^1, \\ 1{\cdot}728 &= 1{\cdot}728 \times 10^0, \\ {\cdot}1728 &= 1{\cdot}728 \times 10^{-1}, \\ {\cdot}01728 &= 1{\cdot}728 \times 10^{-2}, \\ {\cdot}001728 &= 1{\cdot}728 \times 10^{-3}, \text{ and so on.} \end{aligned}$$

In the above list, the number 1·728 is the only one expressed primarily as a value between 1·0 and 9·999, so its common logarithm appears in the table on p. 112; it will be found there that the logarithm of 1728 is 2375, which is to be taken as reading that $\log_{10} 1{\cdot}728 = 0{\cdot}2375$. If this is used as a starting point, the logs of any of the other quantities in the above left-hand column may be found as follows, provided that one remembers that $x^m \times x^n = x^{m+n}$, and that

a logarithm is the power to which 10 must be raised to equal the given number.

$$\begin{aligned} 1728 &= 1{\cdot}728 \times 10^3 \\ 1728 &= 10^{\cdot 2375} \times 10^3 = 10^{\cdot 2375 + 3} \\ &= 10^{3{\cdot}2375}. \\ \therefore \log 1728 &= 3{\cdot}2375. \end{aligned}$$

Again,

$$\begin{aligned} 17{\cdot}28 &= 1{\cdot}728 \times 10^1 \\ &= 10^{\cdot 2375} \times 10^1 = 10^{\cdot 2375 + 1} \\ &= 10^{1{\cdot}2375}. \\ \therefore \log 17{\cdot}28 &= 1{\cdot}2375. \end{aligned}$$

Similarly,

$$\begin{aligned} {\cdot}001728 &= 1{\cdot}728 \times 10^{-3} \\ &= 10^{\cdot 2375} \times 10^{-3} \\ &= 10^{\cdot 2375 + (-3)} \\ &= 10^{\bar{3}{\cdot}2375}. \\ \therefore \log {\cdot}001728 &= \bar{3}{\cdot}2375. \end{aligned}$$

The last example is of considerable importance. The fractional part of the logarithm for this and for all others *must* be kept positive; hence, ·2375 and − 3 (or $\bar{3}$ as it is written) are not added in the usual way. The bar above the 3 ($\bar{3}$) as shown is to indicate that the 3 only is negative, the fractional part remaining positive as stated.

The fractional part of a logarithm is termed the "mantissa", and is always obtained direct from the table; the whole number part, or the "characteristic", as it is called, is obtained by inspection of the number, the mental process being as indicated above. It will be seen that this whole number is always one unit less than the number of individual figures before the decimal point for mixed numbers, and one unit more, but negative, than the number of successive noughts immediately after the decimal point in values less than 1.

It is very important to note that logarithms can only be used for multiplication, division, involution (the finding of roots), and evolution (the raising to powers). If the formulæ or rules under consideration involve the operations of addition and, or subtraction, one or both of these two operations must be performed in the ordinary way.

The following fully worked out examples should be carefully studied, as they demonstrate the most important types of calculation which are adapted for solution by means of logarithms.

Example 24.—The usual way of indicating the equation for the horse-power of a heat engine is—

$$\text{H.P.} = \frac{\text{PLAN}}{33000},$$

where H.P. = the horse-power,
P = the mean effective pressure in lb. per square inch,
L = the length of the stroke in feet,
A = the effective area of the piston in square inches,
N = the number of working strokes per minute,
and 33000 = the number of foot-lb. of work per minute which constitute (as established) 1 horse-power —1 H.P.

The first step in algebraic symbols is to re-write the formula as given in the above form in the way best adapted for calculation by logarithms, i.e.:

$$\text{Log H.P.} = \log P + \log L + \log A + \log N - \log 33000.$$

Notice that in ordinary arithmetical processes PLAN means the *product* of these four terms, and therefore the corresponding logs are *added*; also notice that this product of four terms must be *divided* by 33000 arithmetically, but with logarithms, the log of 33000 is *subtracted* from the sum of the logs in the numerator.

Example 25.—Suppose that numerical values referring to a particular case are as follows:—

P = the mean pressure = 50 lb. per square inch.
L = the length of stroke = 3 ft.
A = the area of piston = 78·54 square inches.
N = the number of working strokes per minute = 380 strokes.

With the usual 33000 ft.-lb. per minute for 1 H.P.
Then, as before:

$$\text{Log H.P.} = \log P + \log L + \log A + \log N - \log 33000.$$
$$= \log 50 + \log 3 + \log 78{\cdot}54 + \log 380 - \log 33000.$$

The logarithms of these numbers must now be found from the table of logarithms, pp. 112 and 113, remembering that the logs given in that table are for numbers between 1 and 10. The operation of finding the various logs is as follows.

To Find Log 50.—$50 = 5 \times 10^1$. The characteristic of the log is therefore 1; the mantissa or fractional part of the log is found from the table. The numbers 10, 11, 12, 13, 14, &c., to 49 in the first column on p. 112, and the numbers 50, 51, 52, 53, &c., to 99 in the first column on p. 113 are really equivalent to 1·0, 1·1, 1·2, 1·3, 1·4, &c., to 4·9; and 5·0, 5·1, 5·2, 5·3, &c., to 9·9 respectively. Each of all the other numbers in the remaining columns on both pp. 112 and 113 is less than 1, but the decimal points are invariably omitted from the tables. In the columns headed 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, each number consists of four digits; but in the columns on the right, headed 1, 2, 3, 4, 5, 6, 7, 8, 9, there are only one or two digits. In the latter cases noughts should be added to the front to make four digits after the decimal point.

Now to obtain log 50. Run down the first column until 50 is reached; this number happens to be at the top of p. 113, and means 5·0 as stated. In the same horizontal line as 50, and under the first figure 0, value 5·00, find 6990. The decimal point in the latter being omitted, the four digits indicate ·6990. The characteristic is 1, as already stated, therefore the complete log of 50 is 1·6990.

To Find Log 3.—3 is a number between 1 and 10; its log is therefore taken directly from the table. Or, to conform strictly to the rule given, $3 = 3 \times 10^0$, whence the characteristic is 0. The mantissa is then obtained by finding 30 (really 3·0) in the left-hand column on p. 112. In the same horizontal row as 30, and under the figure 0 (3·00) find 4771, written ·4771. Log 3 is therefore 0·4771, or simply ·4771.

To Find the Log for the Mixed Number, $78{\cdot}54 = 7{\cdot}854 \times 10^1$. The characteristic is therefore 1. Then referring to the left-hand column of the table on p. 113, look for the 1st and 2nd figures of the four figures of the number 7854, i.e. for 78. Now look for the 3rd figure (5) in the same horizontal row as 78; this value is ·8949, and it indicates the log of 7·85. To find the value for the 4th figure, i.e. 4, the "difference" column on the right-hand side must be consulted. In the same row as ·8949 appears, find the value of the heading 4; this will be found to

be 2. As already stated, noughts must be added to make 4 digits; hence this 2 really means ·0002, and this value must be added to that already found, i.e. 8949. Therefore ·8949 + ·0002 = ·8951. Consequently, the log of 78·54 is 1·8951.

To Find Log 380.—$380 = 3{\cdot}8 \times 10^2$, whence the characteristic is 2. Opposite 38 (3·8), p. 112, and under the figure 0 (3·80) find 5798 (·5798). Log 380 is therefore 2·5798.

Lastly, to Find Log 33000.—$33000 = 3{\cdot}3 \times 10^4$, hence the characteristic is 4. Opposite 33 (3·3), under figure 0 (3·30), find 5185 (·5185). Log 33000 is therefore 4·5185.

The logarithmic form of the calculation indicates that the first four logs are to be added, and from this sum must be subtracted the fifth log. Thus:

Log P = log 50	=	1·6990	add.
,, L = ,, 3	=	·4771	
,, A = ,, 78·54	=	1·8951	
,, N = ,, 380	=	2·5798	
		6·6510	
Log 33000	=	4·5185	subtract.
Whence, log H.P.	=	2·1325	

The number corresponding to the logarithm 2·1325 could be found by inspection of the table of logarithms on pp. 112 and 113, but for 4-figure logarithms it is more usual to use a table of antilogarithms as on pp. 114 and 115. This table differs from the logarithm table solely in arrangement. In the logarithm table, the numbers are given in the left-hand column and in the two heads, while the logarithms appear in the body of the table. In the antilogarithm table, the logarithms are given in the left-hand column and in the two heads, while the numbers corresponding to them are in the body of the table.

To find the number corresponding to log 2·1325. The characteristic is 2, so that the required number is in the form of $N \times 10^2$, where N is a number between 1 and 10. This number N corresponds to the mantissa or fractional part of the logarithm, i.e. to ·1325, and is found as follows: Run down the left-hand column of the antilogarithm table on p. 114 to ·13, and find under the figure 0, 1349. This number is not required, because an 0 does not follow the 3 in 1325. We must move on the same horizontal line as ·13 until we come under the head 2. At this point we find 1355. Now, move further along until we reach the place under the head 5 in the difference column on the right. Here we shall find 2, and this has to be added to the last digit in 1355, thus making it 1357. The decimal point is omitted throughout this table also, and the result 1357 is written 1·357. The number is therefore

$$1{\cdot}357 \times 10^2 = 1{\cdot}357 \times 100,$$

or a value obtained by moving the decimal point in 1·357 two places to the right since the characteristic is 2. Hence the number is 135·7. The H.P. is therefore $1{\cdot}357 \times 10^2 = 135{\cdot}7$.

Beginners are apt to be confused by the apparent similarity of the logarithm and antilogarithm tables, and it is a wise plan to underline, in red, the word antilogarithms at the head of the table as a kind of danger signal.

The operation may appear somewhat involved, but, when proficiency in the use of logs is obtained, the operations become more or less mechanical. At the same time the results obtained are fairly accurate.

When 4-figure logs are used, the results are usually correct to 3 significant figures, a degree of accuracy sufficient for a great number of practical purposes. It need hardly be said, however, that great care must be exercised, not only in getting the correct values, but also in the subsequent processes of addition and subtraction, as a slight mistake in these operations makes, in some cases, a considerable difference in the finished result.

Example 26.—Find the weight of a solid cast-iron roller 18 in. diameter and 78 in. long, if the cast-iron weighs ·263 lb. per cubic inch.

$$\text{Volume V} = \left(\frac{\pi}{4}D^2 l\right) \text{cub. in.}$$

$$\text{Weight W} = \left(\frac{\pi}{4}D^2 l \times {\cdot}263\right)\text{lb.}$$

$$= \frac{3{\cdot}14 \times 18 \times 18 \times 78 \times {\cdot}263}{4}.$$

$$\text{Log W} = \underbrace{\log 3{\cdot}14 + \log 18 + \log 18 + \log 78 + \log {\cdot}263}\; - \underbrace{\log 4}$$

$$\begin{array}{rr} = & {\cdot}4969 \\ & 1{\cdot}2553 \\ & 1{\cdot}2553 \\ & 1{\cdot}8921 \\ & \bar{1}{\cdot}4200 \\ \hline & 4{\cdot}3196 \\ & {\cdot}6021 \leftarrow {\cdot}6021 \\ \hline & 3{\cdot}7175 \end{array}$$

Antilogarithm ·7175 = 5·218.

" 3·7175 = $5{\cdot}210 \times 10^3$

= 5218.

∴ Weight of roller = 5218 lb.

The procedure is similar to that followed in connection with the last example, No. 25. There is, however, one variation. Note that

$$\cdot 263 = 2\cdot 63 \times 10^{-1},$$

since $10^{-1} = \frac{1}{10}$. The characteristic is thus -1, but written $\bar{1}$, as already mentioned; it is only the characteristic that is negative, the mantissa remains positive. In adding the 5 logarithms, the fractional parts are treated as in Example 25. Starting on the right hand we get 6, 9, 1 and 3, the latter representing the unit digit in the sum 23; the 3 is placed next to the decimal point as shown, and the 2, which is $+2$, is carried forward. This 2 added to $1 + 1 + 1 = +5$, and $+5 - 1 = +4$. The whole number is therefore 4, and the full sum is as shown, 4·3196. From this is subtracted the log of 4, i.e. ·6021, and the final value is 3·7175.

Another point worthy of notice is in the treatment of D^2. In the above calculation

$$D^2 = D \times D$$
$$\log D^2 = \log D + \log D,$$
$$\text{i.e.} = 2 \log D.$$

The reasoning is perfectly general, and may be symbolized as under:

$$\text{Log } N^x = x \log N,$$

where x and N are any numbers.

Example 27.—The volume of a sphere is equal to $\frac{4}{3}\pi R^3$. Find the volume in cubic inches of a sphere 12 in. in diameter.

12 in. diameter = 6 in. radius.

$$V = \frac{4 \times 3\cdot 142 \times 6^3}{3}.$$

$$\text{Log } V = \log 4 + \log 3\cdot 142 + 3 \log 6 - \log 3$$

$$\log 4 = \cdot 6021$$
$$\log 3\cdot 142 = \cdot 4972$$
$$3 \log 6 = 3 \times \cdot 7782 = 2\cdot 3346$$
$$3\cdot 4339$$
$$\text{subtract } \log 3 = \cdot 4771$$
$$= 2\cdot 9568$$

$$\text{Antilogarithm } \cdot 9568 = 9\cdot 053$$
$$\text{,, } \quad 2\cdot 9568 = 9\cdot 053 \times 10^2$$
$$= 905\cdot 3.$$
$$\therefore \text{ Volume of sphere} = 905\cdot 3 \text{ cu. in.}$$

An extension of the above principle is demonstrated in the next example.

Example 28.—The horse-power capable of being transmitted by a factory mainshaft is represented by $\frac{D^3N}{100}$, where D is the diameter of the shaft, and N the number of revolutions per minute. Find D, when the H.P. is 1000 and N = 250.

$$\text{H.P.} = \frac{D^3N}{100},$$
$$\therefore D^3N = 100 \text{ H.P.}$$
$$D^3 = \frac{100 \text{ H.P.}}{N}$$
$$\text{and } D = \sqrt[3]{\frac{100 \text{ H.P.}}{N}}$$
$$= \sqrt[3]{\frac{100 \times 1000}{250}}$$
$$= \sqrt[3]{400}.$$
$$\text{Log } D = \tfrac{1}{3} \log 400$$
$$= \tfrac{1}{3} \times 2\cdot 6021$$
$$= \cdot 8674.$$
$$\text{Antilogarithm } \cdot 8674 = 7\cdot 369.$$

The shaft would probably be made $7\frac{1}{2}$ **in. in** diameter.

Example 29.—The diameter of a good quality jute yarn is approximately equal to $\frac{\sqrt{C}}{120}$ where C is the count of the yarn in pounds per spyndle of 14,400 yd. Find the diameter of 14 lb. yarn.

$$D = \frac{\sqrt{C}}{120} = \frac{\sqrt{14}}{120} = \frac{14^{\frac{1}{2}}}{120}.$$

$$\begin{aligned} \text{Log } D &= \tfrac{1}{2} \log 14 - \log 120 \\ &= \tfrac{1}{2} \text{ of } 1\cdot1461 - 2\cdot0792 \\ &= \cdot5731 - 2\cdot0792 \\ &= \bar{2}\cdot4939 \end{aligned} \qquad \begin{array}{r} \cdot5731 \\ 2\cdot0792 \\ \hline \bar{2}\cdot4939 \end{array}$$

$$\begin{aligned} \text{Antilogarithm } \cdot4939 &= 3\cdot118 \\ \text{,,} \qquad \bar{2}\cdot4939 &= 3\cdot118 \times 10^{-2} \\ &= \cdot03118. \end{aligned}$$

$\therefore$ diameter of 14 lb. jute = ·03118 or app. $\frac{1}{32}$ in.

In such a calculation as the above, there is probably only one difficulty for the beginner; that of subtracting 2·0792 from ·5731, as shown on the right-hand side of the answer. The mental process is somewhat as follows:—2 from 11 leaves **9**; 10 from 13 leaves **3**; 8 from 17 leaves **9**; 1 from 5 leaves **4**; 2 from 0, i.e. $0 - 2 = \mathbf{-2}$ or $\bar{2}$. The figures in heavy type are those in the above result.

It is occasionally found necessary to divide such a logarithm as $\bar{2}\cdot4939$ into a number of parts, or again to multiply it by some other number. The examples immediately following illustrate how these operations are carried out; the fundamental principle that the mantissa of the logarithm is always positive, whatever the characteristic may be, must be strictly adhered to.

Example 30.—Find the cube root of ·03118, i.e. find the value of $\cdot03118^{\frac{1}{3}}$.

$$\begin{aligned} \sqrt[3]{\cdot03118} &= \cdot03118^{\frac{1}{3}} \\ \log \sqrt[3]{\cdot03118} &= \tfrac{1}{3} \log \cdot03118 \\ &= \tfrac{1}{3} \times \bar{2}\cdot4939. \end{aligned}$$

The value of $\bar{2}\cdot4939$ as it stands cannot be divided by 3. Add $\bar{1}$ to $\bar{2}$ to obtain $\bar{3}$, which is then divisible without remainder by 3; also add +1 to balance the $\bar{1}$; then we have:

$$\begin{aligned} \frac{\bar{2}\cdot4939}{3} &= \frac{(-3 + 1) + \cdot4939}{3} \\ &= \frac{\bar{3} + 1\cdot4939}{3} \\ &= \bar{1}\cdot4980. \\ \text{Antilog. of } \cdot4980 &= 3\cdot148 \\ \text{,,} \qquad \bar{1}\cdot4980 &= 3\cdot148 \times 10^{-1} \\ &= \cdot3148. \\ \therefore \sqrt[3]{\cdot03118} &= \cdot3148. \end{aligned}$$

The addition of a negative number to the characteristic of such a value as the above is essential, so that the negative value may be divided by the denominator without remainder; of course, a similar positive number must be added to compensate the negative number.

Example 31.—Raise ·03118 to the 5th power, i.e. find the value of $\cdot03118^5$.

$$\begin{aligned} \text{Log } \cdot03118^5 &= 5 \log \cdot03118 \\ &= 5 \times \bar{2}\cdot4939 \\ &= \left\{ \begin{array}{lr} 5 \times \bar{2} & = \overline{10} \\ 5 \times \cdot4939 & = 2\cdot4695 \end{array} \right\} \text{add.} \\ \therefore \log \cdot03118^5 &= \bar{8}\cdot4695. \end{aligned}$$

$$\text{Antilog } \cdot 4695 = 2\cdot 947$$
$$,, \quad \bar{8}\cdot 4695 = 2\cdot 947 \times 10^{-8}$$
$$= \cdot 00000002947.$$
$$\therefore \cdot 03118^5 = \cdot 00000002947.$$

It is worth while noting that if the expression is written as $2\cdot 947 \times 10^{-8}$, that the actual result may be found by moving the decimal point **8** places to the **left**. If the expression had been $2\cdot 947 \times 10^8$, the decimal point would have had to be moved **8** places to the **right**, i.e. the result would have been 294,700,000.

The results obtained by the logarithmic calculations should be compared with the ordinary methods adopted in previous chapters. The former is a most useful method when very large or very small quantities are being dealt with, and it is also a useful check method. The student is advised to check the results in the previous chapters by the methods described in the present chapter.

Exercises, with answers, on p. 98.

CHAPTER VII

TRIGONOMETRICAL RATIOS

That branch of the science of mathematics which treats of the relations existing between the sides and angles of triangles is called **Trigonometry**. In many respects it is one of the most important branches, and, partly because of this importance, and partly because of its wide ramifications, the present chapter is not intended in any way to treat the subject fully, but merely to serve as an introduction to work of a more advanced nature, and to make the young student

familiar with the terms used and the methods employed. Until mentioned otherwise, the remarks will have reference only to acute angles.

DEFINITIONS

Let DCE, fig. 4, be any two straight lines meeting at C and forming an acute angle, i.e. any angle less than 90°. From any point B on the arm or line CE raise a perpendicular meeting the line CD at A, so

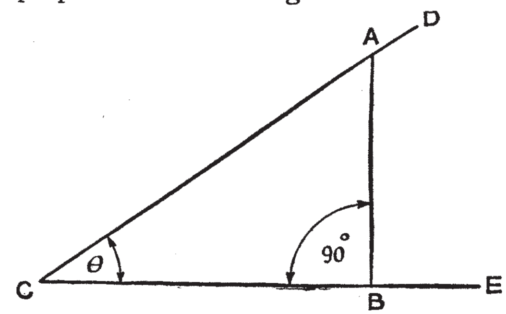


Fig. 4

that the angle ABC is one of 90°—a right angle. Further, let the $\widehat{ACB}$ or $\widehat{DCE}$ measure θ degrees. θ, or theta, is one of the Greek letters, and is often used to denote angles.

The sides of the right-angled triangle ABC may now be named with reference to this angle θ, and as follows:—Side AB is opposite to the angle θ, and hence is called the opposite side, or briefly the opposite. Side CB is adjacent to the angle θ, and is termed the adjacent side, or briefly, the adjacent. Side CA is, obviously, also an adjacent side to the angle θ, but to distinguish it from CB, it is called the hypotenuse, its usual **name**. It is the side opposite to the right angle.

SINE OF AN ANGLE:—The ratio of the side, opposite the given angle, to the hypotenuse is called the sine of the given angle, i.e.

$$\frac{AB}{CA} = \frac{\text{opposite}}{\text{hypotenuse}} = \text{sine of } \widehat{ACB} = \sin\theta.$$

COSINE OF AN ANGLE:—The ratio of the side, adjacent to the given angle, to the hypotenuse is called the cosine of the given angle, i.e.

$$\frac{BC}{CA} = \frac{\text{adjacent}}{\text{hypotenuse}} = \text{cosine of } \widehat{ACB} = \cos\theta.$$

TANGENT OF AN ANGLE:—The ratio of the side, opposite the given angle, to the side adjacent to the given angle is called the tangent of the given angle, i.e.

$$\frac{AB}{BC} = \frac{\text{opposite}}{\text{adjacent}} = \text{tangent } \widehat{ACB} = \tan\theta.$$

It can also be shown that the tangent of an angle is equal to the ratio of the sine to the cosine, i.e.

$$\frac{\sin\theta}{\cos\theta} = \tan\theta.$$

$$\text{Thus, } \sin\theta = \frac{AB}{CA} \text{ and } \cos\theta = \frac{BC}{CA}.$$

$$\therefore \frac{\sin\theta}{\cos\theta} = \frac{\frac{AB}{CA}}{\frac{BC}{CA}} = \frac{AB}{CA} \times \frac{CA}{BC} = \frac{AB}{BC} = \tan\theta.$$

The hypotenuse is necessarily the greatest side of a right-angled triangle. It therefore follows that the ratio

$\frac{\text{opposite}}{\text{hypotenuse}}$, or $\sin\theta$, is always less than 1, except when the two lines coincide.

And, similarly, the ratio:

$\frac{\text{adjacent}}{\text{hypotenuse}}$, or $\cos\theta$, is always less than 1, except when the two lines coincide.

The above three ratios are probably the most common, and are used in a large number of practical problems.

The following ratios are also important:—

COSECANT OF AN ANGLE:—The ratio of the hypotenuse to the side opposite the given angle is called the cosecant of the angle, i.e.

$$\frac{CA}{AB} = \frac{\text{hypotenuse}}{\text{opposite}} = \text{cosecant of } \widehat{ACB} = \text{cosec } \theta.$$

SECANT OF AN ANGLE:—The ratio of the hypotenuse to the side adjacent to the given angle is called the secant of the angle, i.e.

$$\frac{CA}{BC} = \frac{\text{hypotenuse}}{\text{adjacent}} = \text{secant of } \widehat{ACB} = \sec\theta.$$

COTANGENT OF AN ANGLE:—The ratio of the adjacent side to the side opposite the given angle is called the cotangent of the angle, i.e.:

$$\frac{BC}{AB} = \frac{\text{adjacent}}{\text{opposite}} = \text{cotangent of } \widehat{ACB} = \cot\theta.$$

The student should make the following comparisons for himself:—

1. Sine with cosecant.
2. Cosine with secant.
3. Tangent with cotangent.

It would also be instructive to compare the cosecant with the secant.

Example 32. — Construct a right-angled triangle, with sides 3, 4, and 5 in. long, as in fig. 5. Calculate the value of sin A and sin B, cos A and cos B,

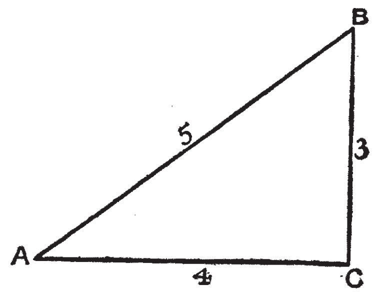


Fig. 5

tan A and tan B, and compare the value of sin A with cos B.

$$\begin{aligned}
\text{Sin A} &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{3}{5} = \cdot 6.\\
\text{Sin B} &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{4}{5} = \cdot 8.\\
\text{Cos A} &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{4}{5} = \cdot 8.\\
\text{Cos B} &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{3}{5} = \cdot 6.\\
\text{Tan A} &= \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC} = \frac{3}{4} = \cdot 75.\\
\text{Tan B} &= \frac{\text{opposite}}{\text{adjacent}} = \frac{AC}{BC} = \frac{4}{3} = 1 \cdot 3.
\end{aligned}$$

Since angle ACB is a right angle, and the sum of the angles in any triangle is equal to 2 right angles, it follows that A + B = 1 right angle. Angles A and B are said to be complementary, i.e. each is the complement of the other.

It is evident from the above results that the following proposition can now be stated. In a right-angled triangle ABC, where C is the right angle,

$$\begin{aligned}
\sin A &= \cos B,\\
\sin B &= \cos A,\\
\tan A &= \frac{1}{\tan B},\\
\tan B &= \frac{1}{\tan A}.
\end{aligned}$$

These four results may be contained in two general statements, as under:

1. In a right-angled triangle, the sine of either acute angle equals the cosine of the other acute angle.

2. In a right-angled triangle, the tangent of either acute angle is the reciprocal of the tangent of the other acute angle.

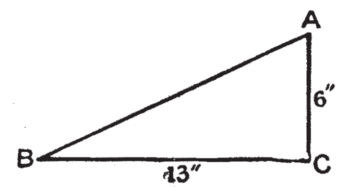


Fig. 6

Example 33.—If 2 sides, BC and AC, of a right-angled triangle (fig. 6), are 13 in. and 6 in. respectively, find the hypotenuse AB, and the value of tan B and cot A; also find sec B and cosec A.

$$\begin{aligned}
\text{Hypotenuse AB} &= \sqrt{(BC^2 + AC^2)}\\
&= \sqrt{13^2 + 6^2}\\
&= \sqrt{169 + 36}\\
&= \sqrt{205}\\
&= 205^{\frac{1}{2}}.\\
\text{Log } 205^{\frac{1}{2}} &= \tfrac{1}{2} \log 205\\
&= \frac{2 \cdot 3118}{2} = 1 \cdot 1559.\\
\text{Antilog } \cdot 1559 &= 1 \cdot 432.\\
\text{Antilog } 1 \cdot 1559 &= 1 \cdot 432 \times 10^1.\\
\therefore \sqrt{205} &= 14 \cdot 32 \text{ in. length of AB.}
\end{aligned}$$

Compare the result with the value obtained by extracting the square root of 205 to 3 places of decimals.

$$\text{Tan B} = \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{13} = 0{\cdot}4616.$$

The logarithmic method, which in this case is the longer one, is as under:

$$\begin{aligned}
\text{Log of tan B} &= \log 6 - \log 13 \\
&= {\cdot}7782 - 1{\cdot}1139 = \bar{1}{\cdot}6643. \\
\text{Antilog } {\cdot}6643 &= 4{\cdot}616. \\
\text{Antilog } \bar{1}{\cdot}6643 &= 4{\cdot}616 \times 10^{-1} = {\cdot}4616. \\
\therefore \text{Tan B} &= {\cdot}4616. \\
\text{Sec B} &= \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{14{\cdot}32}{13} = 1{\cdot}1015 \text{ or } 1{\cdot}102 \text{ approx.} \\
\text{Log of sec B} &= \log 14{\cdot}32 - \log 13 \\
&= 1{\cdot}1559 - 1{\cdot}1139 = {\cdot}0420. \\
\text{Antilog } {\cdot}0420 &= 1{\cdot}102. \\
\therefore \text{Sec B} &= 1{\cdot}102, \text{ or, as above, } 1{\cdot}1015. \\
\text{Cosec A} &= \frac{\text{hypotenuse}}{\text{opposite}} = \frac{14{\cdot}32}{13} = 1{\cdot}102.
\end{aligned}$$

It will now be seen why the sine and co-sine, the tangent and co-tangent, and the secant and co-secant are so named.

TRIGONOMETRICAL RATIOS OF CERTAIN ANGLES.—Certain angles, such as 30°, 45°, 60°, 90°, &c., are more in evidence than others for educational work, particularly because those named form very simple and useful fractions of a complete circle. Their trigonometrical ratios are important, and in many cases worth committing to memory. They are considered in the following section:—

ANGLE OF 0°.—Referring to fig. 7, let $\widehat{ECB}$ be

any small angle. From B draw BA perpendicular to CE, so that BAC is a right-angled triangle. Further, let CB, which is a radius of the circle, be the unit of length, and suppose it equal to 1.

$$\text{Sin D} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{AB}{1}.$$

Now as the radius CB is made to move clockwise towards the radius CE, AB becomes smaller and

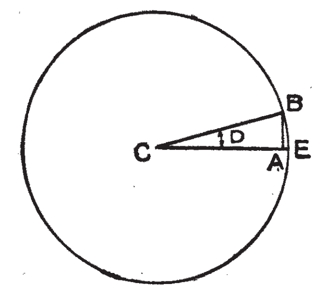


Fig. 7

smaller, until when CB coincides with CE, angle D becomes 0°; then AB becomes 0, so that

$$\sin 0^\circ = \frac{0}{1} = 0.$$

$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{CA}{CB} = \frac{CA}{1}.$$

Now as CB is made to move clockwise towards CE, CA becomes greater and greater, until when CB again coincides with CE, angle D becomes 0°, and CB = CA = 1, so that

$$\cos 0^\circ = \frac{1}{1} = 1.$$

It was previously shown, p. 50, that

$$\frac{\sin A}{\cos A} = \tan A,$$

$$\text{whence } \tan A = \frac{0}{1} = 0.$$

By employing similar methods, the student may work out for himself the values of sec 0°, cosec 0°, and cot 0°.

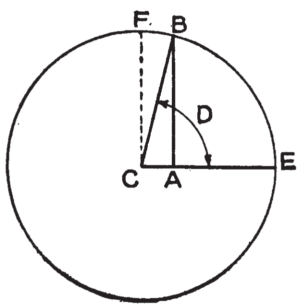


Fig. 8

ANGLE OF 90° (RIGHT ANGLE).—Let $\widehat{BCE}$ or D (fig. 8) be an angle a little less than 90°. From B draw BA perpendicular to CE, so that $\widehat{BAC}$ is a right-angled triangle. Further, as in the last example, let CB be the unit of length and equal to 1.

$$\text{Sin D} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AB}{CB} = \frac{AB}{1}.$$

Now as CB is made to move counter-clockwise towards CF, AB becomes greater and greater, until, when CB coincides with CF, angle D becomes 90°, and AB = CF = 1, so that

$$\sin 90° = \frac{1}{1} = 1.$$

$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{CA}{CB} = \frac{CA}{1}.$$

Now as CB is made to move counter-clockwise towards CF, CA becomes smaller and smaller, until,

when CB coincides with CF, angle D becomes 90°, and CA becomes 0, so that

$$\cos 90° = \frac{0}{1} = 0.$$

$$\text{Again, } \frac{\sin D}{\cos D} = \tan D.$$

$$\therefore \tan 90° = \frac{1}{0} = \infty, \text{ or infinity.}$$

NOTE.—The sign ∞ is used to express "infinity". If the value 1 be divided by a smaller quantity, such as $\frac{1}{2}$, the result is 2. If the divisor is made smaller, say $\frac{1}{4}$, the result is 4, and so on. As the divisor decreases, the quotient becomes larger and larger, until when the divisor becomes very small indeed and closely approaching 0, the quotient is an exceedingly large quantity. The latter condition would obtain whatever number appeared in the numerator, and the quotient cannot be expressed in figures, hence the use of the term "infinity" and its sign ∞.

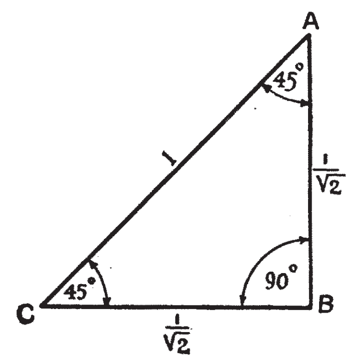


Fig. 9

ANGLE OF 45°.—Let ABC, fig. 9, be a right-angled triangle with the side AB = side BC. It is evident (and can be proved by Euclid I. 5) that the angles at A and C are equal to each other, and since B is a right angle, angles A and C must each equal 45°. Also, let AC be the unit of length and equal to 1.

Now, in the right-angled triangle ABC, the square on the hypotenuse is equal to the sum of the squares on the other two sides, i.e.:

$$
\begin{aligned}
(AC)^2 &= (AB)^2 + (BC)^2 \\
&= 2(AB)^2 \text{ since } AB = BC. \\
\therefore\ 1^2 &= 2(AB)^2, \\
\text{and } 2(AB)^2 &= 1 \\
(AB)^2 &= \tfrac{1}{2} \\
\therefore\ AB &= \sqrt{\tfrac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}. \\
\therefore\ AB &= BC = \frac{1}{\sqrt{2}}.
\end{aligned}
$$

$$\text{Sin C} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\frac{1}{\sqrt{2}}}{1} = \frac{1}{\sqrt{2}}.$$

$$\text{Cos C} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{\frac{1}{\sqrt{2}}}{1} = \frac{1}{\sqrt{2}}.$$

$$\text{Tan C} = \frac{\text{opposite}}{\text{adjacent}} = \frac{AB}{BC} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1,$$

$$\text{or tan C} = \frac{\sin C}{\cos C} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1.$$

NOTE: $\frac{1}{\sqrt{2}} = 0{\cdot}7071$ correct to 4 places of decimals, but as $\sqrt{2}$ is incommensurable, $\frac{1}{\sqrt{2}}$ is also incommensurable, and in many cases it is convenient to use the $\frac{1}{\sqrt{2}}$ expression, which is correct, in preference to the decimal value, which is only approximately correct.

ANGLE of 60°.—Let ABC, fig. 10, be an equilateral triangle with sides AB, BC, and CA each equal to 1. In any triangle, the sum of the three angles = 180°, and in an equilateral triangle all the angles are equal, so that each angle is $\frac{180}{3} = 60°$.

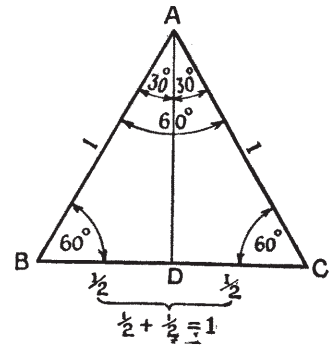


Fig. 10

From A, draw AD perpendicular to BC. $\widehat{ADB}$ is now a right-angled triangle in which D is a right angle, and angle B = 60°. Further, the point D bisects the side BC, so that BD = CD = ½.

$$
\begin{aligned}
\text{Then } (AB)^2 &= (AD)^2 + (BD)^2, \\
\text{so that } (AD)^2 &= (AB)^2 - (BD)^2, \\
\text{whence } AD &= \sqrt{(AB)^2 - (BD)^2} \\
&= \sqrt{1^2 - (\tfrac{1}{2})^2} \\
&= \sqrt{1 - \tfrac{1}{4}} = \sqrt{\tfrac{3}{4}} \\
&= \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}.
\end{aligned}
$$

$$\text{Sin B} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}.$$

$$\text{Cos B} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{\frac{1}{2}}{1} = \frac{1}{2}.$$

$$\text{Tan B} = \frac{\text{opposite}}{\text{adjacent}} = \frac{AD}{BD} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3};$$

$$\text{or tan B} = \frac{\sin B}{\cos B} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}.$$

ANGLE OF 30°.—Referring to fig. 10 above, it is evident that if angle D is 90°, and B is 60°, $\widehat{DAB}$ must be 30°, since 90° + 60° + 30° = 180°.

$$\text{Sin } 30° = \sin \widehat{DAB} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{\frac{1}{2}}{1} = \frac{1}{2}.$$

$$\text{Cos } 30° = \cos \widehat{DAB} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}.$$

$$\text{Tan } 30° = \tan \widehat{DAB} = \frac{\text{opposite}}{\text{adjacent}} = \frac{BD}{AD} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}.$$

$$= \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}},$$

$$\text{or } \tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}.$$

If the results obtained with angles of 60° and 30° are compared, they will be found in complete accord with the general statements on p. 53, i.e. in a right-angled triangle:—

(1) $\text{Sin } 60° = \cos 30° = \sqrt{\frac{3}{2}}.$

(2) $\text{Cos } 60° = \sin 30° = \frac{1}{2}.$

(3) $\left.\begin{aligned}\text{Tan } 60° &= \sqrt{3}\\ \text{Tan } 30° &= \frac{1}{\sqrt{3}}\end{aligned}\right\}$ reciprocals.

The following rule is perfectly general: In all cases, the sine of an angle equals the cosine of its complement; and in all cases the tangent of an angle is the reciprocal of the tangent of its complement.

NOTE.—Two angles are said to be complements of each other when their sum equals 90°.

The above results are often very useful, and it

will be convenient to have them in a table for ready reference.

Fraction of circle ...	0	$\frac{1}{12}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$
Angle in degrees ...	0	30	45	60	90
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

By various methods, which do not come within the scope of an elementary work, the values of the sine, cosine, tangent, &c., have been worked out for all angles.[1] These values are tabulated for ready reference, and sine tables, &c., as they are called, are most useful for the solution of problems in practice. When practically absolute values are desired, the tables mentioned in footnote should be consulted, but for the great majority of practical calculations, four-figure tables, giving the sines, &c., of all angles, up to 90°, in steps of 1 minute, are amply sufficient. Again, it is not absolutely necessary to have separate tables for each of the ratios, since all the ratios are interdependent, and may, by suitable arrangement, be written in terms of the sine and the tangent. Hence it is only necessary, except for special purposes, to have tables of sines and tangents. Such tables are given on pp. 116 to 123.

It has already been shown that sin 30° = cos 60°.

[1] See Blackie's *Handy Book of Logarithms*.

This relation may be expressed in general terms. Thus:

$$\text{Cos D} = \sin(90 - D)$$

Again, it has been shown that the secant of an angle is equal to the reciprocal of the cosine of the angle, i.e.

$$\sec D = \frac{1}{\cos D},$$

$$\text{but } \frac{1}{\cos D} = \frac{1}{\sin(90 - D)}.$$

$$\therefore \sec D = \frac{1}{\sin(90 - D)}.$$

Further, it has been shown that the cosecant of an angle is equal to the reciprocal of the sine of the angle, i.e.

$$\text{cosec } D = \frac{1}{\sin D}.$$

Lastly, it has been shown that the cotangent of an angle is the reciprocal of its tangent, i.e.

$$\cot D = \frac{1}{\tan D}.$$

Hence, by using tables of the sines and tangents only, and by using the above relations, it is possible to obtain the six chief trigonometrical ratios.

The examples immediately following indicate how the tables are used. It should be noted that special forms of the sine and tangent called Logarithmic Sines and Tangents are sometimes used, and that the tables referred to are tables of Natural Sines, &c., so-called to distinguish them from the logarithmic form.

Example 34.—Find the value of sin 18° 29′.

Referring to the Natural Sines table on pp. 116–7, it will be seen that the extreme left-hand column contains all the complete angles from 0° to 90°, and that the column headed 0′ contains the corresponding sines. The body of the table shows the sines of the intermediate angles, the intervals or steps being 6 min., while the right-hand columns show the amounts to be added or subtracted for differences of 1, 2, 3, 4, or 5 min.

One should proceed as follows to find the sine of 18° 29′: Find 18 in the left-hand column, and in the same horizontal line and under the heading 0′, find 3090. This is really ·3090, but the decimal point is omitted throughout because no sine exceeds 1. Hence, ·3090 is the sine of 18° 0′. Now, move along the same horizontal row and find 3156 under the heading 24′; ·3156 is thus the sine of 18° 24′. This angle differs from 18° 29′ by 5′; under the heading 5′ in the table of differences find 14; this 14 really means ·0014, only the significant figures being given. The value ·0014 is to be added to ·3156, making ·3170. The sine of 18° 29′ is therefore ·3170.

An alternative, and perhaps simpler, method is as follows: First find the sine of 18° 30′; this is ·3173. Then take off the sine of 1′ to obtain the sine of 18° 29′. In the difference column find 3, or ·0003 for 1′, and subtract ·0003 from ·3173; we thus obtain ·3170 as before.

Example 35.—Find the value of cos 63° 8′.

$$\text{Cos } 63^\circ 8' = \sin(90 - 63^\circ 8')$$
$$= \sin 26^\circ 52'.$$

Looking over the table we find sin 26° 54′ to be ·4524. The difference for 2′ is 5, i.e. ·0005, which must be subtracted from ·4524, giving ·4519.

Alternatively, find sin 26° 48′, which is ·4509. The difference for 4′ is 10, i.e. ·0010, which must be added to ·4509, giving ·4519 as before. Therefore, cos 63° 8′ = ·4519.

Example 36.—Find the value of tan 43° 19′.

From the table of natural tangents, tan 43° is found to be ·9325. Tan 43° 18′, in the same row, is ·9424. The difference for 1′ is 6 or ·0006, therefore ·9424 + ·0006 = ·9430. Tan 43° 19′ is, therefore, ·9430.

The above examples show how to find the sine, &c., corresponding to a given angle. The same tables are also used for finding the angle, given the value of the sine or tangent. Thus:

Example 37.—Find the angle which has a sine = ·9167.

The method of using the table for this purpose consists primarily in locating the sine, and then seeing to which angle it corresponds. Thus, with the example given, an inspection of the column 0′ shows two consecutive sines corresponding to ·9135 and ·9205. The former value is less, and the latter value greater, than ·9167; the two values inspected represent angles of 66° and 67° respectively, so that the actual angle required is between these two. An inspection of the 66° horizontal line shows ·9164 under the 24′ head, so that ·9164 is the sine of 66° 24′. But this sine differs from that given by ·0003. A glance at the different figures in the same 66° line shows 3 under the heading 3′, so that the required angle is 3′ greater than 66° 24′, i.e. it must be 66° 27′.

Example 38.—Find the angle corresponding to a tangent of 1·1654. By inspecting the 0′ column in the Natural Tangent Table, two consecutive values occur which, being 1·1504 and 1·1918, are respectively less and greater than the given value. The angle is therefore greater than 49°, but less than 50°. It must thus be found in the 49° line. An inspection of this line reveals two consecutive values less and greater than 1·1654, the given value. These two consecutive values are 1626 and 1667 under the headings 18′ and 24′ respectively. The required angle is thus greater than 49° 18′, but less than 49° 24′. The difference between ·1626 and ·1654 is ·0028, and 28 is found in the difference column under the heading 4′. Consequently, the required angle is 49° 18′ + 4′ = 49° 22′.

In all cases where the figures in the difference column do not coincide exactly with the difference value required, the nearest value is taken; the result is then correct to the nearest minute.

LOGARITHMIC SINES, TANGENTS, &c.—Reference to a table of sines or cosines will show that the value is never greater than 1; the characteristic of their logarithms is thus always negative. For example, the sine of 45° is ·7071, and log ·7071 = $\bar{1}$·8495. In working out calculations involving sines, &c., these negative values prove very inconvenient. The inconvenience, however, may be avoided by using tables of Logarithmic Sines, &c.

A logarithmic sine table gives directly the logarithm of the sine of an angle with one slight variation. All the values given have a characteristic of 10 added to them in order to avoid the use of a negative characteristic. Thus, if one inspects a table for the logarithmic sine of 45°, it is found to be 9·8495. The actual value has been shown to be $\bar{1}$·8495, but $\bar{1}$·8495 + 10·0000 = 9·8495. In general:

$$\text{L} \sin A = 10 + \log \text{ of } \sin A,$$

where L sin indicates logarithmic sine.

The same remarks hold good for logarithmic tangents.

With the exception noted above, the method of using the logarithmic sine and tangent tables (these two being mostly used) is exactly the same as that demonstrated in reference to the tables of natural sines and tangents, and no difficulty is experienced in their use, except, perhaps, on one point. This difficulty will be understood after a study of the two following examples.

Example 39.—Find from the table L sin 20° 38′. Find 20 (i.e. 20°) in the left-hand column. Under the head 0′ find 9·5341, which is L sin 20° 0′. Move along the line to the right and find 5463 (i.e. ·5463) under the head 36′; 9·5463 is thus L sin 20° 36′. In the difference columns find 7 (i.e. ·0007) under the head 2′. Now 9·5463 + ·0007 = 9·5470, and this = L sin 20° 38′.

Example 40.—Find from the table L tan 84° 59′. Find 84 (i.e. 84°) in the left-hand column, and under the head 0′ find 10·9784, which is L tan 84° 0′. Move along the line to the right and find $\bar{0}494$ under the head 54′. The bar (¯) over the 0 in $\bar{0}494$ signifies that the integer or whole number part of the logarithmic tangent must be 1 more than that already found, i.e. that L tan 84° 54′ is 11·0494 (and not 10·0494). In the difference column and under the head 5′ find 66 (i.e. ·0066), which must be added to 11·0494, giving 11·0560 as L tan 84° 59′.

It is important to remember that the bar is a mere sign-post to call attention to the change in the integer of the characteristic, and must not be confused with the somewhat similar position of the bar over a characteristic figure. In the latter case, as already indicated, the bar is a negative sign.

Relations between the Angles and Sides of Triangles.—Many important relations between the angles and the sides of a triangle can be established by the principles of geometry. The formulæ given in the following paragraphs can all be definitely proved correct. These proofs, however, are of little value to the student at this stage, and it is therefore proposed to omit such proofs in the present elementary treatment of the subject. The results, however, are of great value, and an effort will be made in the concluding section to show how they can be used in the solution of practical problems.

1. The sides of a triangle are proportional to the sines of the angles opposite them.

If ABC be any triangle, having the sides *a*, *b*, and *c* opposite the angles A, B, and C respectively, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

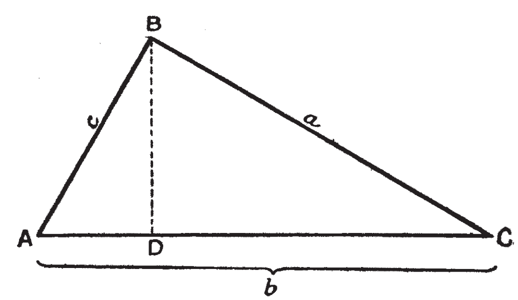


Fig. 11

2. Any side of a triangle is equal to the sum of the projections of the two other sides of the triangle upon it.

If ABC (fig. 11) be any triangle, AD is the projection of the side AB on AC, and DC is the projection of the side BC on AC. AD + DC = AC, the third side.

Consider now the following with respect to the angle A:

$$\frac{\text{AD}}{c} = \frac{\text{adjacent}}{\text{hypotenuse}} = \cos \text{A}.$$

$$\text{Now if } \cos \text{A} = \frac{\text{AD}}{c},$$
$$c \cos \text{A} = \text{AD}.$$

Similarly, it may be shown that

$$a \cos \text{C} = \text{DC},$$
$$\text{hence, } \underbrace{\text{AD} + \text{DC}} = c \cos \text{A} + a \cos \text{C},$$
$$\text{i.e. } \quad \underbrace{\text{AC}} = c \cos \text{A} + a \cos \text{C},$$
$$\text{or } \quad b = c \cos \text{A} + a \cos \text{C}.$$

In the same way it may be proved that

$$c = a \cos \text{B} + b \cos \text{A},$$
$$\text{and that } a = b \cos \text{C} + c \cos \text{B}.$$

3. It is sometimes necessary to be able to express the sine or cosine of any angle in a triangle in terms of the sides only.

By an adaptation of Euclid II, 13 and 14, and with reference to fig. 11 it can be proved that

$$\underbrace{\text{BC}^2}_{a^2} = \underbrace{\text{AB}^2}_{c^2} + \underbrace{\text{AC}^2}_{b^2} - 2\,\underbrace{\text{AC}}_{b}\cdot\text{AD};$$

but since AD = c cos A (see above)—

$$a^2 = c^2 + b^2 - 2\,bc \cos \text{A},$$
$$\therefore \cos \text{A} = \frac{c^2 + b^2 - a^2}{2\,bc}.$$

Consequently, we have:

$$1.\ \cos \text{A} = \frac{b^2 + c^2 - a^2}{2\,bc}.$$
$$2.\ \cos \text{B} = \frac{c^2 + a^2 - b^2}{2\,ca}.$$
$$3.\ \cos \text{C} = \frac{a^2 + b^2 - c^2}{2\,ab}.$$

4. In a somewhat similar manner, the value of the sine can be expressed in terms of the sides. It can be proved that

$$1.\ \sin \text{A} = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}.$$
$$2.\ \sin \text{B} = \frac{2}{ca}\sqrt{s(s-a)(s-b)(s-c)}.$$
$$3.\ \sin \text{C} = \frac{2}{ab}\sqrt{s(s-a)(s-b)(s-c)}.$$

In each case ABC is the triangle having sides a, b, and c opposite the angles A, B, and C respectively, and s is the semi-perimeter of the triangle, i.e.

$$\frac{a + b + c}{2}.$$

The following progressive examples illustrate the application of these rules and formulæ to the solution of concrete problems.

Example 41.—Triangle ABC, fig. 12, represents to scale 3 forces acting in one

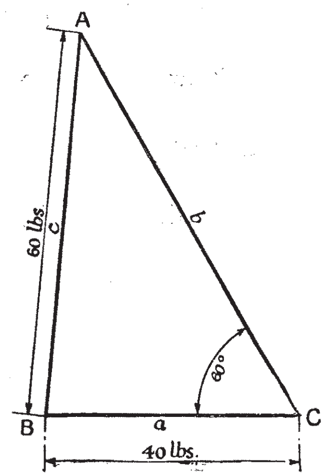


Fig. 12

plane. If AB and BC represent forces of 40 lb. and 60 lb. respectively, and angle C is 60°, find the force AC and the angles A and B.

$$\text{Since } \frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{40}{\sin A} = \frac{60}{\sin C}$$

$$60 \sin A = 40 \sin 60°$$

$$\sin A = \frac{40 \sin 60°}{60}$$

$$= \frac{40 \times \cdot 8660}{60} = \cdot 5773;$$

$$\text{so that angle } A \doteqdot 35°\ 15' \text{ or } 35°\ 16'.$$

$$\text{Now, } A + B + C = 180°,$$

$$\text{and } B = 180° - (C + A)$$

$$= 180° - (60° + 35°\ 15')$$

$$= 180° - 95°\ 15'.$$

$$\therefore \text{angle } B = 84°\ 45'.$$

$$\text{Now, } \frac{b}{\sin B} = \frac{c}{\sin C},$$

$$\text{so that } \frac{b}{84°\ 45'} = \frac{60}{\sin 60°},$$

$$\text{whence } b = \frac{60 \sin 84°\ 45'}{\sin 60}$$

$$= \frac{60 \times \cdot 9958}{\cdot 8660}.$$

$$\text{Log } b = \log 60 + \log \cdot 9958 - \log \cdot 8660$$

$$= 1 \cdot 7782 + \bar{1} \cdot 9981 - \bar{1} \cdot 9375$$

$$= 1 \cdot 8388.$$

$$\therefore b = 6 \cdot 900 \times 10^1 = 69.$$

$$\text{Hence force } b = 69 \text{ lb.}$$

Example 42.—In a flax spreader (spread-board) the reach is 40 in., and the difference in levels between the centre of the feed or retaining roller and the centre of the front or drawing roller is 10 in. Find

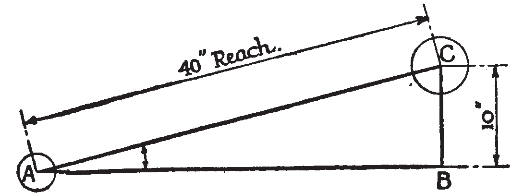


Fig. 13

the horizontal distance between the rollers, and the angle BAC, see fig. 13.

$$\text{Sin } A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{10}{40} = \frac{1}{4}.$$

$$\therefore A = 14°\ 29'.$$

$$\text{Tan } A = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{AB},$$

$$\text{i.e. } \tan 14°\ 29' = \frac{10}{AB}.$$

$$\therefore \cdot 2583 = \frac{10}{AB},$$

$$\text{whence } \cdot 2583\ AB = 10$$

$$\text{or } AB = \frac{10}{\cdot 2583}.$$

$$\text{Log } AB = \log 10 - \log \cdot 2583$$

$$= 1 \cdot 0000 - \bar{1} \cdot 4121$$

$$= 1 \cdot 5879.$$

$$\therefore AB = 3 \cdot 872 \times 10^1 = 38 \cdot 72.$$

$$\text{Hence A to B} = 38 \cdot 72 \text{ in.}$$

The above question could be solved as under:

Since the triangle is right-angled

$$40^2 = 10^2 + (AB)^2$$
$$(AB)^2 = 40^2 - 10^2$$
$$AB = \sqrt{40^2 - 10^2}$$
$$= \sqrt{1600 - 100}$$
$$= \sqrt{1500}.$$
$$\text{Log } AB = \tfrac{1}{2}\log 1500$$
$$= \frac{3\cdot1761}{2} = 1\cdot5881.$$
$$\therefore AB = 38\cdot74 \text{ in.}$$

The different methods give practically identical

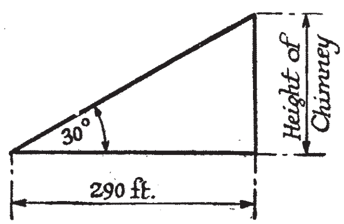


Fig. 14

results, it having been pointed out that 4-figure logarithms give results true to 3 significant figures only.

Example 43.—An observer standing on the same level as, and at a distance of 290 ft. from, the base of a factory chimney, finds by means of a theodolite that the lines leading to the base and to the top of the chimney subtend an angle of 30°, see fig. 14. Find the height of the chimney.

$$\text{Tan } 30^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{height}}{290 \text{ ft.}},$$
$$\text{i.e. } \tan 30^\circ = \frac{h}{290},$$
$$\text{whence } h = \tan 30^\circ \times 290.$$
$$\text{Log } h = \text{L} \tan 30^\circ + \log 290 - 10$$

(10 is subtracted because 10 is added to log tan 30°).

$$\therefore \log h = 9\cdot7614$$
$$2\cdot4624$$
$$12\cdot2238$$
$$10\cdot0000$$
$$2\cdot2238$$

$$\therefore h = 1\cdot674 \times 10^2 = 167\cdot4 \text{ ft.}$$

Example 44.—In setting out rope-pulley grooves, see fig. 15, it is often required to know the distance from the centre of the rope to the point where the converging lines, viz. the sides of the groove, would meet, i.e. the distance x. If the rope-groove angle is θ degrees, and the radius of the rope r inches, find the distance x. Then use the resulting equation to find x to the nearest $\frac{1}{32}$ in., when θ is 45°, and the diameter of the rope is $1\frac{3}{4}$ in.

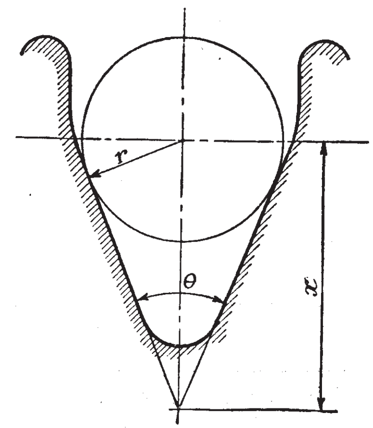


Fig. 15

Draw r perpendicular to one of the lines of the groove, fig. 15, then:

$$\frac{r}{x} = \sin\frac{\theta}{2}.$$

$$\therefore x = \frac{r}{\sin\frac{\theta}{2}}.$$

Introducing the above given values we have

$$x = \frac{\frac{1\frac{3}{4}}{2}}{\sin\frac{45^\circ}{2}} = \frac{\frac{7}{8}}{\sin 22\frac{1}{2}^\circ}$$

$$= \frac{7}{8 \times 0\cdot3827} = \frac{7}{3\cdot0616}$$

$$= 2\tfrac{9}{32} \text{ in. to nearest 32nd.}$$

Exercises, with answers, on p. 100.

CHAPTER VIII

YARN COUNTS, ETC.

Systems of Counting Yarns.—There are two distinct systems of counting yarns: (i) that in which the count represents the weight of a certain fixed length, and (ii) that in which the count represents the length contained in a fixed weight.

First System.—The table used in the jute, hemp, and heavy flax yarn trade may be taken as typical of this system. In this table, the fixed length is termed the "spyndle", and is equal to 48 cuts or leas of 300 yd. each, or 14,400 yd. The weight of 1 spyndle of yarn in pounds avoirdupois is the count of the yarn. Thus, 3 lb. yarn is that yarn of which 14,400 yd. weigh 3 lb.

Second System.—The cotton yarn table may be regarded as typical of the second system. In this system the fixed weight is 1 lb., and the length is measured in hanks of 840 yd. each. The number of

hanks of 840 yd. each in 1 lb. of any yarn is the count of that yarn. Thus 16^s cotton implies that 16 hanks of 840 yd. each weigh 1 lb.

Variation of Counts.—In a 3 lb. flax yarn, 14,400 yd. weigh 3 lb. In a 6 lb. flax yarn, 14,400 yd. weigh 6 lb. Thus, the fixed length of yarn in the second case weighs twice as much as the same fixed length of yarn in the first case. 6 lb. yarn is thus heavier and thicker than 3 lb. yarn, as might be expected from the names of the counts themselves. The inference is that when the count of a yarn is expressed as a variable weight for a fixed length of yarn, the larger the number representing the count, the greater is the sectional area of the yarn.

Again, 16^s cotton means that 16 hanks of 840 yd. each, or 13,440 yd., weigh 1 lb. Now, 32^s cotton means that 32 hanks of 840 yd. each, or 26,880 yd., weigh 1 lb. Hence, it takes twice the length of the second yarn to weigh the same as the first. The inference here is that the larger the count number, the smaller is the sectional area of the yarn.

Twisted or Folded Yarns.—Yarns are folded or twisted together for various reasons, chief among which are development of strength, uniformity, durability, and the attainment of coloured effects.

When yarns are thus folded or twisted, it is usually desired to express the counts in such a way as to indicate the composition of the yarn. For example, a jute twist may be called 3-ply 8 lb. yarn, commonly written $3/8^s$, and implying that the twist or compound thread has been formed by combining together, by a process of twisting, 3 individual threads of jute yarn, each of which weighs 8 lb. per spyndle. Secondly, a folded cotton yarn may be marked $8/60^s$, indicating that the twist or compound yarn is made up of

8 threads of 60^s cotton. Lastly, and this method should be carefully compared with the last definition, silk twist may be termed 60/3, or 60^s 3-fold, indicating that 3 threads have been twisted together to form a twist or compound thread, the count of the twist thread being 60^s.

EQUIVALENT COUNTS.—Many fabrics are composed of mixtures of yarns of various fibres, and as there is at least one special system of counting yarns spun from each kind of fibre, it follows that cloth calculations would become extremely involved were it not possible to express the counts of each constituent yarn on one common basis. This end is obtained by the method of using equivalent counts, e.g. cotton counts expressed on the same basis as woollen, worsted, silk, raw silk, jute, or linen.

From a knowledge of the basis of the count number, rules may be deduced which enable any count of one system to be expressed as the equivalent of some yarn count in any other system. The following examples illustrate how this may be done:

Example 45.—Express 8 lb. per spyndle (jute yarn count) in yards per pound.

$$8 \text{ lb. per spyndle} = 8 \text{ lb. per } 14{,}400 \text{ yd.}$$

$$\therefore \frac{14{,}400 \text{ yd.}}{8 \text{ lb.}} = 1800 \text{ yd. per pound.}$$

Symbolically, N lb. per spyndle may be expressed as yards per pound, thus:

$$N \text{ lb. per spyndle} = \frac{14{,}400}{N} \text{ yd. per pound.}$$

Example 46.—Express 8 leas per pound (linen count) as yards per ounce (1 lea = 300 yd.).

$$\begin{aligned} 8 \text{ leas per pound} &= (8 \times 300) \text{ yd. per } (1 \times 16) \text{ oz.} \\ &= 2400 \text{ yd. per } 16 \text{ oz.} \\ \therefore \frac{2400 \text{ yd.}}{16 \text{ oz.}} &= 150 \text{ yd. per ounce.} \end{aligned}$$

Symbolically,

$$\begin{aligned} N \text{ leas per lb.} &= (N \times 300) \text{ yd. per } (1 \times 16) \text{ oz.} \\ &= 300\,N \text{ yd. per } 16 \text{ oz.} \\ &= \frac{300\,N}{16} \text{ yd. per ounce} \\ &= \frac{75\,N}{4} \text{ yd. per. ounce} \\ &= 18{\cdot}75\,N \text{ yd. per ounce.} \end{aligned}$$

From the above one may deduce the following rule. To change linen counts (leas per pound) to yards per ounce, multiply the count number by 18·75.

$$N \text{ leas per pound} = 18{\cdot}75\,N \text{ yd. per ounce.}$$

Example 47.—Convert 20^s cotton yarn count to Galashiels woollen count.

Cotton count = number of hanks of 840 yd. each in 1 lb.

Galashiels count = number of cuts of 300 yd. each in $1\frac{1}{2}$ lb.

$$\begin{aligned} 20^s \text{ cotton} &= (20 \times 840) \text{ yd. per pound} \\ &= (20 \times 840) \text{ yd. in } 16 \text{ oz.} \\ &= \frac{20 \times 840}{16 \text{ oz.}} \text{ yd. per ounce} \\ &= \frac{20 \times 840 \times 24}{16} \text{ yd. in } 24 \text{ oz.} \\ &= \frac{20 \times 840 \times 24}{300 \times 16} \text{ cuts in } 24 \text{ oz.} \\ &= 84 \text{ cuts in } 24 \text{ oz } (1\tfrac{1}{2} \text{ lb.}) \\ &= 84^s \text{ Galashiels woollen count.} \end{aligned}$$

In symbols:

Let C = cotton count.

G = Galashiels woollen count.

$$G = \frac{C \times 840 \times 24}{16 \times 300} \text{ cuts in 24 oz.}$$

$$= 4 \cdot 2\ C.$$

The rule may be put into words thus: To convert cotton counts to equivalent Galashiels woollen counts multiply the cotton counts by 4·2.

Example 48.—Express 8 lb. jute in equivalent linen count.

Jute counts = lb. per spyndle of 14,400 yd.

Linen counts = No. of leas (300 yd. each) in 1 lb.

8 lb. per spyndle = 8 lb. per 14,400 yd.

$$\therefore \frac{14{,}400 \text{ yd.}}{8 \text{ lb.}} = 1800 \text{ yd. per pound,}$$

$$\text{and } \frac{1800 \text{ yd. per pound}}{300 \text{ yd. per lea}} = 6 \text{ leas per pound.}$$

∴ 8 lb. per spyndle = 6 leas per pound (linen count).

The above exhibits the general method, applicable to all cases. The following is a shorter method, and is applicable only to those cases where the unit length is common. Thus, the cut for jute is the same length as the lea for linen. The American cut is also the same.

$$\frac{14{,}400 \text{ yd. per spyndle}}{300 \text{ yd. per lea}} = 48 \text{ leas or cuts per spyndle.}$$

If the count of jute yarn is J lb. per spyndle, there will be 48 leas in J lb. Consequently,

$$\frac{48 \text{ leas}}{J} = L \text{ leas per pound.}$$

That is to say, if J = the jute count in pounds per spyndle, and L = linen counts in leas per lb.,

$$L = \frac{48}{J},$$

and from this it follows that

$$JL = 48,$$

$$\text{and } J = \frac{48}{L}.$$

Example 49.—Convert 40^s linen to equivalent cotton count.

Linen counts = No. of leas (300 yd. each) in 1 lb.

Cotton counts = No. of hanks (840 yd. each) in 1 lb.

40^s linen = (40 × 300) yd. per lb.

= 12,000 yd. per lb.

$$\text{and } \frac{12{,}000 \text{ yd.}}{840 \text{ yd. per hank}} = \frac{100}{7} = 14\tfrac{2}{7} \text{ hanks per pound, the cotton count.}$$

Generally, if C = cotton counts,

L = linen counts,

$$C = \frac{L \times 300}{840} = \frac{L}{2 \cdot 8},$$

$$\text{that is, } C = \frac{L}{2 \cdot 8},$$

$$\text{whence } L = 2 \cdot 8\ C.$$

Example 50.—Convert 16^s cotton into the equivalent woollen count in the Yorkshire skein system.

Cotton counts = No. of hanks (840 yd. each) in 1 lb.

Yorkshire skein = No. of skeins (256 yd. each) in 1 lb.

16^s cotton = (16 × 840) yd. per pound
= 13,440 yd. in 1 lb.

$$\therefore 16^s \text{ cotton} = \frac{13{,}440 \text{ yd.}}{256 \text{ yd. per skein}} = \frac{105}{2}$$

= 52·5 Yorkshire skein woollen.

Generally, if C = cotton counts,
Y = Yorkshire skein.

$$Y = \frac{C \times 840}{256} = 3\cdot28125\ C.$$

$$Y = \text{say } 3\cdot28\ C,$$

$$\text{whence } C = \frac{Y}{3\cdot28}.$$

When absolute accuracy is required, it is wise to use the expression $\frac{C \times 840}{256}$ instead of the approximate decimal equivalent.

Example 51.—Change 20^s Yorkshire skein woollen count to its equivalent in the raw silk system.

Yorkshire skein woollen = 1 yd. per dram for No. 1.
16 yd. per ounce for No. 1,
256 yd. per pound for No. 1.

Raw silk counts = yards per ounce.

20^s Yorkshire skein woollen = 20 × 256 yd. per pound

$$= \frac{20 \times 256}{16} \text{ yd. per ounce}$$

= 320 yd. per ounce raw silk count.

The Dewsbury heavy woollen district system is also yards per ounce. Hence:

20^s Yorkshire skein = 320 raw silk = 320 Dewsbury.

Generally, if Y = Yorkshire skein counts,
S_r = raw silk counts,
D = Dewsbury counts,

$$S_r = \frac{Y \times 256}{16}. \qquad D = \frac{Y \times 256}{16}.$$

Since $S_r = D$,

$$\therefore S_r = 16\ Y \quad \text{and} \quad D = 16\ Y,$$

$$\text{whence } Y = \frac{S_r}{16} \quad \text{and} \quad Y = \frac{D}{16}.$$

Example 52.—Convert 30^s worsted count into the equivalent cotton count.

Worsted count = No. of hanks (560 yd. each) in 1 lb.
Cotton count = No. of hanks (840 yd. each) in 1 lb.
30^s worsted = (30 × 560) yd. per pound
= 16,800 yd. per pound.

$$\therefore 30^s \text{ worsted} = \frac{16{,}800}{840} = 20^s \text{ cotton.}$$

Generally, if C = cotton counts,
W = worsted counts,

$$W = \frac{C \times 840}{560} = \frac{3}{2}\ C,$$

$$W = 1\tfrac{1}{2}\ C,$$

$$\text{whence } C = \frac{2\ W}{3}.$$

RESULTANT COUNTS.—When two or more threads of the same or of different counts are folded or twisted together, the count of the folded or twisted yarn is termed the resultant count. The resultant count may be nominal or actual, depending on whether the contraction in length due to twisting has been neglected or considered.

Nominal Resultant Count of 2-ply Yarns.—If one thread of 16^s cotton is twisted with a second thread of 16^s cotton, the twist thread, as previously shown, is $2/16^s$. Now 16^s cotton means 16 hanks in 1 lb., and, if no allowance is made for contraction or take-up due to the twisting operation, 16 hanks of twist will contain:

16 hanks of 16^s cotton	=	1 lb.
16 ,, 16 ,,	=	1 lb.
16 hanks of twist	=	2 lb.

$$\text{whence, } \frac{16 \text{ hanks of twist}}{2 \text{ lb.}} = 8 \text{ hanks per pound,}$$

so that the nominal resultant count is 8^s cotton.

Generally, when the count is expressed as a variable length for a fixed weight, and the yarns composing the twist are alike in count, the nominal resultant count is the quotient of the individual count and the number of compounded threads.

If N = the count of the single yarn,
p = the number of plies or threads,
R_n = the nominal resultant count,

$$R_n = \frac{N}{p}.$$

Again, if 1 thread of 8 lb. jute is twisted with a second similar thread, the twist thread is 2/8. If 1 spyndle of each yarn is twisted, no allowance being made for take-up, due to twisting, 1 spyndle of twist will contain:

1 spyndle of 8 lb. yarn	=	8 lb.
1 ,, 8 ,,	=	8 lb.
1 spyndle of $2/8^s$ twist	=	16 lb.

so that the nominal resultant count is 16 lb. per spyndle.

Generally, when the count is expressed as a variable weight for a fixed length, the nominal resultant count is the product of the count of the yarn (when all the individual yarns are the same count) and the number of plies. Using the same notation as before: $R_n = Np$.

When the constituent threads of the folded or twisted yarn are of different counts, the above two rules are not applicable. To take an example, suppose one thread of 16^s cotton is to be folded or twisted with one thread of 32^s cotton. In 32 hanks of the twist there will be, if contraction due to twist is neglected:

32 hanks of 32^s cotton	=	1 lb.
32 ,, 16^s ,,	=	2 lb.
32 hanks of twist	=	3 lb.

$$\text{whence } \frac{32 \text{ hanks}}{3 \text{ lb.}} = 10\tfrac{2}{3} \text{ hanks per lb.,}$$

the nominal resultant count.

This solution may be obtained in another way. If 32 lb. of 16^s cotton = 512 hanks, are twisted with 512 hanks of 32^s cotton $= \frac{512}{32} = 16$ lb., the resulting 512 hanks of twist will weigh $32 + 16 = 48$ lb., whence

$$\frac{512 \text{ hanks}}{48 \text{ lb.}} = 10\tfrac{2}{3} \text{ hanks per lb.,}$$

the nominal count.

Notice that the arithmetical work involved in the above is the same as the following:

$$\frac{32 \times 16}{32 + 16} = \frac{512}{48} = 10\tfrac{2}{3} \text{ count.}$$

That is to say, when the counts of the yarn are expressed as variable lengths for a fixed weight, the nominal resultant count of two dissimilar yarns is equal to the quotient of the product and sum of the respective counts.

If C_1 is the count of one yarn,
C_2 is the count of the other yarn,
R_n is the nominal resultant count,

$$R_n = \frac{C_1 \times C_2}{C_1 + C_2}.$$

In the case of yarns, such as jute, where the count is expressed as a variable weight for a fixed length, there is less difficulty than is experienced in the fixed weight system. Thus, suppose a 10 lb. yarn is to be twisted with an 8 lb. yarn, the resulting spyndle of twist yarn, neglecting contraction due to twist, will obviously weigh 10 lb. + 8 lb. = 18 lb. per spyndle.

From the above it may be deduced that in such cases, the nominal resultant count is equal to the sum of the constituent counts, and this holds good for any number of plies. That is to say, if the counts of the individual yarns are

$$C_1, C_2, C_3 \ldots\ldots\ldots C_n$$
$$R_n = C_1 + C_2 + C_3 + \ldots\ldots C_n.$$

Resultant Counts of Twist Yarns of more than 2 Plies.—As indicated in the last paragraph, no difficulty is experienced with multiple plies when the individual yarns are expressed as variable weights of a fixed length. But, when the counts express variable lengths for a fixed weight, as in cotton, spun silk, woollen, worsted, and linen, the reasoning is not quite so simple.

Example 53.—Suppose that a twist thread is to be

composed of 1 thread each of 20^s, 30^s, and 40^s cotton, the resulting nominal count can be found as under:

40 hanks of	40^s	cotton	=	1 lb.
40 ,,	30^s	,,	=	$1\frac{1}{3}$ lb.
40 ,,	20^s	,,	=	2 lb.
40 hanks of twist			=	$4\frac{1}{3}$ lb.

whence, $\dfrac{40 \text{ hanks twist}}{4\frac{1}{3} \text{ lb.}} = \dfrac{120}{13} = 9\frac{3}{13}$ hanks per lb.,

which is the nominal resultant count.

From this numerical example, a general rule may be deduced. Notice that the highest count is taken as a starting-point, not because it is absolutely necessary, but merely because it is satisfactory and convenient; so far as the result is concerned, any of the counts concerned would serve equally well; indeed, any number, whole or fractional, would give the correct result.

Suppose several cotton yarns, of which the counts are respectively C_1, C_2, $C_3 \ldots\ldots C_n$ are to be twisted together, then:

$$\begin{array}{llll} C_1 \text{ hanks of } C_1 \text{ yarn} & = \dfrac{C_1}{C_1} \text{ lb.} \\ C_1 \quad ,, \quad C_2 \quad ,, & = \dfrac{C_1}{C_2} \text{ lb.} \\ C_1 \quad ,, \quad C_3 \quad ,, & = \dfrac{C_1}{C_3} \text{ lb.} \\ \vdots & \\ C_1 \quad ,, \quad C_n \quad ,, & = \dfrac{C_1}{C_n} \text{ lb.} \\ \hline C_1 \text{ hanks of twist} & = \left(\dfrac{C_1}{C_1} + \dfrac{C_1}{C_2} + \dfrac{C_1}{C_3} + \ldots \dfrac{C_1}{C_n}\right) \text{lb.,} \end{array}$$

whence the nominal resultant count is:

$$\frac{C_1}{\frac{C_1}{C_1} + \frac{C_1}{C_2} + \frac{C_1}{C_3} + \cdots \frac{C_1}{C_n}} = \frac{C_1}{1 + \frac{C_1}{C_2} + \frac{C_1}{C_3} + \cdots \frac{C_1}{C_n}}.$$

The rule thus found may be used to check the example worked out by arithmetical methods, where $C_1 = 40^s$, $C_2 = 30^s$, $C_3 = 20^s$. Then:

$$R_n = \frac{C_1}{1 + \frac{C_1}{C_2} + \frac{C_1}{C_3}}$$

$$= \frac{40}{1 + \frac{40}{30} + \frac{40}{20}} = \frac{40}{\frac{60 + 80 + 120}{60}}$$

$$= \frac{40}{1 + 1\frac{1}{3} + 2} \text{ or } \frac{40}{\frac{260}{60}}$$

$$= \frac{40}{4\frac{1}{3}} \text{ as before, or } \frac{40 \times 60}{260} = \frac{120}{13}$$

$$= \frac{120}{13} = 9\tfrac{3}{13}{}^s \text{ count as before.}$$

Many different forms of these calculations may be found, as reference to such works as *Calculations and Structure of Fabrics*, by Woodhouse and Milne, and *Yarn Counts and Calculations*, by Themhija, will show, but the mathematical principles underlying the various forms do not differ. Two other examples only will, therefore, be demonstrated, since they will illustrate practical points.

YARN COUNTS TO PRODUCE REQUIRED TWIST COUNTS.—In certain cases, it may be required to find what count of yarn should be twisted with a known count in order to produce a given count of twist. Suppose, therefore, that it is desired to make 8^s cotton twist by using 1 thread of 20^s cotton and one other thread, what count should the second thread be?

It is shown on p. 84, that when two yarns are twisted together,

$$R_n = \frac{Cc}{C + c},$$

$$\text{hence, } Cc = R_n(C + c)$$

$$= R_nC + R_nc,$$

$$\text{i.e. } Cc - R_nc = R_nC,$$

$$c(C - R_n) = R_nC.$$

$$\therefore c = \frac{R_nC}{C - R_n} \text{ or } \frac{CR_n}{C - R_n}.$$

In the example given, $R_n = 8^s$, and $C = 20^s$; it is required to find c.

$$c = \frac{CR_n}{C - R_n} = \frac{20 \times 8}{20 - 8}$$

$$= \frac{160}{12} = 13\tfrac{4}{12} \text{ or } 13\tfrac{1}{3}{}^s.$$

This result may be checked by the rule previously found. If $13\frac{1}{3}{}^s$ yarn and 20^s yarn are twisted together, the nominal resultant count should be 8^s. Thus:

$$R_n = \frac{Cc}{C + c} = \frac{20 \times 13\frac{1}{3}}{20 + 13\frac{1}{3}} = \frac{20 \times 13\frac{1}{3}}{33\frac{1}{3}}$$

$$= \frac{20 \times 40 \times 3}{3 \times 100} = 8^s \text{ cotton.}$$

CONTRACTION.—Experience in the twisting, doubling, or folding of yarns will naturally give those interested the opportunity of observing the actual amount of contraction which occurs when yarns are twisted together. These results should be filed for reference, and use may be made of them to deduce other contractions.

For example, suppose it is required to find what two counts of cotton yarn should be twisted together to give a twist thread equal in count to 14^s, allowing 10 % for shrinkage in length,

let C = the required counts, then,

100 hanks of C + 100 hanks of C = 90 hanks of 14^s twist, since 100 % − 10 % = 90 %.

$$\therefore \frac{100}{C} + \frac{100}{C} = \frac{90}{14},$$

$$\text{i.e. } \frac{200}{C} = \frac{90}{14}.$$

$$90\,C = 14 \times 200.$$

$$\therefore C = \frac{14 \times 200}{90}$$

$$= 31\tfrac{1}{9} \text{ hanks per lb., or } 31\tfrac{1}{9}{}^s.$$

The result may be checked in the following manner: The nominal resultant count will be $\frac{31\frac{1}{9}}{2}$. If, as stated, these two yarns shrink 10 % when twisted, the actual resultant count will be 10 %, or $\frac{1}{10}$ less than the nominal, i.e. it will be

$$\frac{31\frac{1}{9}}{2} - \frac{10}{100} \text{ of } \frac{31\frac{1}{9}}{2}$$

$$= \frac{280}{9 \times 2} - \left(\frac{1}{10} \times \frac{280}{9 \times 2}\right)$$

$$= \frac{140}{9} - \left(\frac{1}{10} \times \frac{140}{9}\right)$$

$$= \frac{140}{9} - \frac{14}{9} = \frac{126}{9} = 14^s \text{ cotton.}$$

The general case may be stated as follows: Let the yarn counts be C and c respectively; let R_a = the actual resultant count, and r = the rate per cent contraction. Then,

$$\frac{100}{C} + \frac{100}{c} = \frac{100 - r}{R_a}.$$

$$R_a\left(\frac{100}{C} + \frac{100}{c}\right) = 100 - r.$$

$$R_a = \frac{100 - r}{\frac{100}{C} + \frac{100}{c}}.$$

This result shall be tested by

Example 54.—Suppose that 20^s yarn is to be twisted along with 70^s yarn, and that there is 10 % contraction.

$$R_a = \frac{100 - 10}{\frac{100}{20} + \frac{100}{70}}$$

$$= \frac{90}{5 + 1\frac{3}{7}} = \frac{90}{6\frac{3}{7}}$$

$$= \frac{90 \times 7}{45} = 14^s \text{ twist yarn (actual count).}$$

Again, the actual resultant count thus found may be checked as in the previous example, thus:

$$R_n = \frac{Cc}{C + c} = \frac{70 \times 20}{70 + 20}$$

$$= \frac{1400}{90}$$

$$= \frac{140}{9}.$$

Then, if the contraction is 10 per cent or $\frac{1}{10}$, the actual resultant count will be:

$$R_a = \frac{140}{9} - \frac{10}{100} \text{ of } \frac{140}{9}$$
$$= \frac{140}{9} - \frac{14}{9}$$
$$= \frac{126}{9} = 14^s.$$

Exercises, with answers, on p. 103.

EXERCISES

Chapter I, pp. 1–7

1. A sample of cloth, 2 sq. in. in area, weighs 8·9 gr. If the cloth is 30 in. wide, what is its weight in ounces per yard? (7000 gr. = 1 lb.)
Ans. 10·985 oz. per yard.

2. A warp thread withdrawn from a sample of cloth, 3 in. long, is found, when straightened, to measure 3·2 in. What length must the warp be laid to yield 60 yd. of cloth?
Ans. 64 yd.

3. A weft thread, withdrawn from the sample of cloth mentioned in Exercise 2, is 4 in. long, while the cloth is $3\frac{3}{4}$ in. wide. What is the probable width of the warp in the reed if the cloth is to be 52 in. wide?
Ans. 55·47 in. in reed.

4. The horse-power required to drive spinning frames of the same spindle pitch varies as the number of spindles, and as the square of the speed of the spindles in r.p.m. If 70 ring spindles running at 8500 r.p.m. require 1 horse-power, what power will be needed to drive a ring frame of 280 spindles running at 7500 r.p.m? *Ans.* 3·114 H.P.

5. A spinning mill of 35,000 spindles uses 60 tons of coal per week of 55 hours. Trade circumstances necessitate a partial stoppage of production, obtained by

stopping $\frac{1}{5}$ of the spindles, and running the remainder 40 hours per week. What is now the probable coal consumption? *Ans.* 34·91 tons.

6. A fine roving frame, producing 4^s roving with 2·40 turns per inch, has a front roller, $1\frac{1}{8}$ in. diameter, running at 141 r.p.m., while the flyers make 1200 r.p.m. The production per spindle per day of 9 hours under these conditions is 1·971 lb. What production should be expected from the same frame making 6^s roving, the turns or twist on the roving being 2·92 per inch? *Ans.* 1·08 lb.

7. A cotton mule, spinning 20^s, gives a production of 1·75 lb. per spindle per week of 56 hours. How long will it take a 20,000 spindle mill, running 50 hours a week, to produce material for a 50-ton order? *Ans.* 3·584 weeks.

8. If ring frames, under the same conditions as in Exercise 7, can give $2\frac{1}{2}$ lbs. per spindle, how long should a 20,000 ring spinning mill take to complete the order? *Ans.* 2·5088 weeks.

9. Three partners in a worsted spinning mill invest £10,000, £8000, and £3000 in the concern. If the net profits in a particular year are £1890, what share of profits should each receive for investment only? *Ans.* £900. £720. £270.

10. The speeds of toothed wheels geared together are in inverse proportion to the numbers of teeth they contain. A hemp-softener driving pulley runs at 240 r.p.m., and the pulley pinion of 17 teeth drives a wheel of 52 teeth on the shaft of the lower softening roller. Find the speed of this roller in r.p.m. *Ans.* 78·46 r.p.m.

11. A ring spinning mill contains the following machinery; the values on the right indicate the horse-power required to drive the various groups.

	H.P.
(1) PREPARING MACHINERY:	
Bale-breakers, openers, scutchers, &c. ...	275
Cards, drawings, slubbings, intermediates, and rovings	382
(2) SPINNING MACHINERY:	
75 352-spindle warp ring frames ... ... } 65 372-spindle weft ,, ,, ... }	670
(3) WINDING MACHINERY:	
Drum, pirn and warp winders, reels, &c. ...	20
(4) WARPING MACHINERY:	
Warping and slasher sizing machines ...	40
(5) WEAVING MACHINERY:	
1346 looms (32 in. to 72 in. reed space) ...	456
(6) FINISHING MACHINERY:	
Cloth folders, presses, &c.	35
Total horse-power	1878

Find the following:

(1) The ratio between the number of spinning spindles and the total horse-power. *Ans.* 26·93 to 1.

(2) The ratio between the number of looms and the total horse-power. *Ans.* 1 to 1·395.

(3) The ratio between the number of spinning spindles and the number of looms. *Ans.* 37·58 to 1.

Chapter II, pp. 8-13

1. Five wrappings of cotton yarn are taken from a skip and weighed separately: the weights are 25, 26, 27, $25\frac{1}{2}$, and $26\frac{1}{2}$ gr. What is the average count of the yarn in hanks of 840 yd. per pound, a wrapping being 120 yd.? *Ans.* $38{\cdot}46^s$ count.

2. What is the average count of weft used in a cotton-weaving shed of 400 looms when different wefts are being used in the following proportions:—80 looms on 25^s,

70 looms on 20^s, 50 looms on 30^s, 100 looms on 35^s, 60 looms on 40^s, and 40 looms on 45^s? *Ans.* $31\frac{1}{2}^s$.

3. A warp contains 2400 threads, and is made of different yarns in the following proportions:—$\frac{5}{12}$ of 40^s cotton, $\frac{1}{3}$ of 30^s cotton, and $\frac{1}{4}$ of 20^s cotton. What is the average count of the complete warp? *Ans.* $31 \cdot 67^s$.

4. In a 9-bale batch of jute, each weighing 400 lb., there are 2 bales at £62, 2 at £60, 2 at £58, 2 at £64, and 1 at £52, prices per ton. Find the average cost per ton. *Ans.* £60 per ton average.

Chapter III, pp. 13–18

1. The delivery roller of a drawing-frame is $2\frac{5}{8}$ in. diameter, and runs at 120 r.p.m. What will be the production of the frame in hundredweights per day of 9 hours, if the sliver weighs 4 lb. per 100 yd., the frame has 6 deliveries, and the production time is 85 % of the actual time occupied? *Ans.* 27·02 cwt.

2. A 54-in. cotton-reeling machine has provision for 40 hanks, and the reel runs at 120 r.p.m. Find the number of hanks (840 yd. each) it will reel in a 9-hour day, the production factor being 56 %. *Ans.* 2592 hanks.

3. A 90-in. reel runs at 75 r.p.m. and has 24 spindles. Find the length of yarn in spyndles of 14,400 yd. it will reel per day of 9 hours. If the actual turn-off is 70 spyndles, what is the percentage of production time? *Ans.* $168\frac{3}{4}$ spyndles. 41·48 %.

4. An oil pump used for supplying oil to the main bearings of a factory engine has a stroke of $2\frac{1}{4}$ in. and a bore of $1\frac{1}{2}$ in., while it makes 400 working strokes per minute. Find the quantity of oil which is pumped into the bearings in gallons per minute, given that the efficiency of the pump is 60 %, and that a cubic foot of oil = $6\frac{1}{4}$ gallons. *Ans.* 3·45 gall.

5. In a weaving shed, driven electrically on the group system, 58 looms, from 37 in. to 56 in. reed space, are driven by a 100 H.P. motor running at 485 r.p.m. If the motor pulley is 15 in. diameter, and the driving-shaft of the looms is to run at 150 r.p.m., find the diameter of the driven pulley, 2 % being allowed for slip on the driven pulley. *Ans.* $49\frac{1}{2}$ in. diameter.

6. 600 threads of 8 lb. jute are dressed and starched. If the weight on the beam is 320 lb., and it is known that the dressed yarn contains 8 % of starch or size, what length of warp is on the beam? (Jute counts = weight in pounds per spyndle of 14,400 yd.) *Ans.* 888·8 yd. $888\frac{8}{9}$ yd.

7. A textbook contains 6 questions in each of 20 chapters; if a student works out the answers for 84 of the questions, what percentage of the whole has he done? *Ans.* 70 %.

8. The flax industry of the United Kingdom gives employment to about 29,760 males, and 70,720 females. Find the ratio of males to females, and the percentage of each employed.
Ans. 1 to 2·376 or 93 males to 221 females; 29·62 % males, 70·38 % females.

9. 9 bales of jute, each 400 lb., have an average cost of £60 per ton. The fibre is spun into yarn weighing 8 lb. per spyndle of 14,400 yd., and there is a loss of 5 % in working it over the machinery. The yarn is then sold at 6*s*. per spyndle, less 4 %; what does the spinner receive for it, and what is the price of the yarn per ton? *Ans.* £123, 2*s*. $4\frac{4}{5}$*d*. £80, 12*s*. $9\frac{1}{2}$*d*.

10. If the cost of spinning the above yarn is £16 per ton, what is the spinner's total profit on the transaction, the profit per ton, and the percentage profit?
Ans. £7, 1*s*. 8*d*. £4, 12*s*. $9\frac{1}{2}$*d*. 5·753 %.

Chapter IV, pp. 18–24

1. 5 leas of cotton yarn weigh 250 gr., and the yarn is known to contain 6 per cent of added moisture. What is the count of the yarn in hanks of 840 yd. each per pound (the cotton count), and what is the original count, 1 lea being 120 yd.? *Ans.* 20^s. Original count $21 \cdot 2^s$.

2. A sample of cotton yarn weighing 1 lb. tested in a conditioning oven is found to lose 20 dr. What percentage of moisture has been removed? *Ans.* 7·81 per cent.

3. A sample pound of cotton, when dried in an oven, loses 6 per cent of its weight; what is its dry weight? *Ans.* 240·64 dr.

4. A bale of Brazilian cotton weighs 220 lb., and a sample pound taken from the bale loses 25 dr. in the conditioning oven. Find the conditioned weight of the bale, allowing a regain of $8\frac{1}{2}$ per cent. *Ans.* 215·39 lb.

5. A quantity of cotton in the dry condition weighs 2 tons; what weight of water must be added if the allowable regain is $8\frac{1}{2}$ per cent, and what percentage of moisture will be in the cotton after conditioning?
Ans. 380·8 lb. 7·83 per cent.

Chapter V, pp. 24–30

1. Two grades of cotton worth 20*d.* and 14*d.* respectively are to be blended to form a mixture valued at 16*d.* per pound. What quantities of each should there be in 1000 lb. of the blend? *Ans.* $333\frac{1}{3}$ lb. at 20*d.*; $666\frac{2}{3}$ lb. at 14*d.*

2. If a further blend worth 16*d.* per pound is required, and cotton worth 18*d.* per pound is to replace the above grade at 20*d.*, how many pounds of the 14*d.* variety will be required in 1000 lb. of the mixture?
Ans. 500 lb. at 18*d.*; 500 lb. at 14*d.*

3. 500 lb. of shoddy at 15*d.* per pound, and 500 lb. of cotton at 12*d.* per pound are blended; what is the value per pound of the mixture? *Ans.* $13\frac{1}{2}$*d.*

4. Three colours of wool are mixed to obtain a desired shade; if there are 100 lb. of black yarn at 3*s.* 6*d.* per pound, 24 lb. of white yarn at 3*s.* per pound, and 6 lb. of red yarn at 3*s.* 4*d.* per pound, what is the average cost per pound of the blend?
Ans. 3·4 shillings. Approximately 3*s.* $4\frac{3}{4}$*d.*

5. Five bales of jute, each weighing 400 lb., are batched or blended; they are worth respectively, £60, £56, £50, £49, and £40 per ton. What is the cost per pound and per ton of the mixture, and the total value of the blend?
Ans. $5\frac{13}{28}$*d.* or 5·46*d.* £51. £45, 10*s.* $8\frac{4}{7}$*d.*

6. A batching mixture is composed of 3 qualities of oil costing 1*s.*, 1*s.* 6*d.*, and 2*s.* per gallon, and mixed in the proportion of 3, 2, and 1 respectively. What is the cost per gallon of the mixture? *Ans.* 1*s.* 4*d.* per gallon.

7. One ton of linen yarn is made from a mixture of two classes of fibre. If 20 per cent of the fibre is lost in the process, how many pounds were there to begin with? See also No. 11 below. *Ans.* 2800 lb.

8. Two tons of shoddy at 1*s.* 4*d.* per pound have to be mixed with a proportion of cotton at 1*s.* 1*d.* per pound. What quantity of cotton should be added to make a mixture worth 1*s.* 2*d.* per pound?
Ans. 8960 lb. cotton at 1*s.* 1*d.*

9. Wool costing 5*s.* per pound is mixed with cotton at 1*s.* 6*d.* per pound, so that the blend will sell at 2*s.* 9*d.* per pound, and allow 10 per cent profit. Find the proportion of wool and cotton in the blend.
Ans. $28\frac{4}{7}$ per cent wool. $71\frac{3}{7}$ per cent cotton.

10. A batch for the manufacture of shop twine consists of 80 per cent hemp, and 20 per cent jute. The hemp

costs £75 per ton, and the jute £50 per ton. Find the average cost of the mixture and the quantities required for a 10 ton order.

Ans. £70. 8 tons hemp. 2 tons jute.

11. 20 cwt. of linen yarn is to be made from a mixture of two classes of material costing 2*s.* and 3*s.* per pound respectively. What quantity of each must be taken so that the average value will be 2*s.* 9*d.* per pound, assuming that the waste in manufacture is 20 per cent, and that the waste so made is sold for 5*d.* per pound?

Ans. $2006\frac{2}{3}$ lb. at 2*s.* $793\frac{1}{3}$ lb. at 3*s.*

Chapter VI, pp. 30–48

For logarithmic solution.

1. Find the approximate diameter of 30^s linen yarn, given that:—$d = \frac{1}{16\sqrt{c}}$, where d = diameter, and c = count in leas of 300 yd. per pound.

Ans. ·01141. Approximately $\frac{1}{88}$ in.

2. The expansion of a steel shaft in inches equals ·00000672 LR, where L is the nominal length of the shaft in inches, and R is the range of temperature in degrees Fahrenheit to which the shaft is subjected. Find the actual amount of expansion in the case of a factory shaft, 200 ft. long, subjected to a change of temperature of 40° F.

Ans. ·6452 in.

3. In a jute cloth, the length of warp in spyndles of 14,400 yd. required per piece is $\frac{ptrl}{26,640}$, where p = the porter, t = the threads per split, r = the reed width in inches, and l = the laid length of the warp in yards. Find the spyndles of warp required for a 12 porter cloth, 4 threads per split, $47\frac{1}{2}$ in. reed width, and 108 yd. laid length.

Ans. 9·243 spyndles.

4. In a certain cloth, the length of weft, in spyndles of 14,400 yd. per piece, is $\frac{src}{14,400}$, where s = number of shots per inch, r = reed width in inches, and c = yards of cloth in one piece. Find the spyndles of weft required when there are 9 shots per inch, $28\frac{1}{2}$ in. reed width, and 115 yd. of cloth.

Ans. 2·047 spyndles.

5. The front roller of a cotton-drawing frame is $1\frac{3}{4}$ in. diameter, and runs at 300 r.p.m. Find the calculated production in pounds per day of 9 hr., if the sliver delivered weighs 100 gr. per yard.

Ans. 353·4 lb.

6. If the cop mentioned in Example 61, and illustrated in fig. 41, Part I, weighs 238·9 gr., and contains 1076 yd. of cotton yarn, find the count of the yarn (hanks of 840 yd. per pound), and also find the approximate diameter of the yarn.

Ans. $37{\cdot}53^s$, probably 36^s cotton count.
Diameter = ·00823″ = approximately $\frac{1}{120}$ in.

7. Given that the approximate diameter of a cotton yarn is $\frac{1}{k\sqrt{c}}$, where k is a constant depending upon the condition of the yarn, and c is the count in hanks per pound, find k for a mule-spun cotton cop, using the results obtained in Exercise 6 above.

Ans. $d = \frac{1}{19{\cdot}76\sqrt{c}}$.

8. A Lancashire boiler is 30 ft. long and 9 ft. diameter, inside measurements; it has two flues running through it, the outside diameter of each being 3 ft. 5 in. Find the volume occupied by the two flues, and subtract it from the inside volume of the boiler to find the maximum volume of water it can hold. (Log. 3·1416 = ·4971.)

Ans. 1358 cu. ft.

9. The transverse stiffness of a shaft varies directly as the fourth power of the diameter, and inversely as the load and the cube of the length. Express this in a mathe-

matical formula, and use the expression to find the relative stiffness of the following:

(*a*) A shaft 10 in. diameter, 11 ft. long, and a total load of 6 tons;

(*b*) A shaft 11 in. diameter, 13 ft. long, and carrying 7 tons.

Ans. $S \propto \frac{d^4}{Wl^3}$. (*a*) 1·252. (*b*) ·9523.

10. The approximate length of a belt in roll form is given by the formula $L = \cdot 1309N(D + d)$ where L = length in feet, N = the number of coils in the roll, D = the outside diameter of the roll in inches, and d = the diameter of the hole in inches. Find the length of a belt of 25 coils, 5 in. hole, and $17\frac{1}{2}$ in. outside diameter.

Ans. 73·62 ft.

Chapter VII, pp. 48–74

1. The upper surface—the race—of the lay of a loom dips towards the centre; if this dip is $\frac{1}{2}$ in. in a 100 inch reed space loom, find the difference between the straight-line distance between the shuttle-boxes or the ends of the lay (i.e. a chord), and the actual line of the race (an arc).

Ans. 102·1 in. − 100 in. = 2·1 in.

2. A vertical warping mill is to be 12 yd. in circumference, and to have 28 spokes. (The plan of the mill would be a 28-sided polygon.) Find the effective length of the spars carrying the 28 upright spokes.

Ans. 11·47 ft. or 11 ft. $5\frac{3}{4}$ in.

3. A linen or jute yarn reel is to be 90 in. in circumference and to have 12 spars. Find the effective length of the spokes carrying the spars. *Ans.* 28·98 in.

4. A cotton yarn reel or swift is to accommodate 54 in. hanks, and to have 8 spars (the usual number is 6). Find the effective length of the spokes carrying the spars.

Ans. 17·637 in.

5. Draw any right-angled triangle ABC. Measure each side as accurately as possible, and calculate therefrom the value of the sine, cosine, and tangent of each of the acute angles. Use tables of sines and tangents to find out the sizes of the angles, and check the results by measurement with a protractor.

6. A so-called perfectly balanced plain cloth has the structure indicated in fig. 16, and is to be made from 10^s cotton, the diameter of which is $\frac{1}{26 \cdot 2\sqrt{c}}$, where c is the count (number of hanks of 840 yd. each in 1 lb.). Find

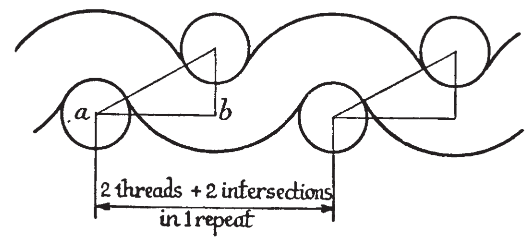


Fig. 16

the yarn diameter, and calculate the number of threads per inch to be used in the cloth, if the horizontal distance ab between each pair of threads equals $\sqrt{3}$ times the diameter of the yarn.

Ans. $\frac{1}{83 \cdot 84}$. 48·4 threads per inch.

7. A twilled cloth, $\frac{5\ 4}{3\ 3}$ (see Fig. 17), is required to be made from 10^s cotton. Find the number of threads per inch in the cloth when the distance $ab = \sqrt{3}$ the diameter d of the yarn, and distance between the centres of adjoining threads = d. *Ans.* 70·15 threads per inch.

8. The top of a roving flyer tapers from $1\frac{3}{16}$ in. diameter to $1\frac{1}{16}$ in. diameter in a length of $1\frac{3}{4}$ in. Calculate the included angle of the taper. *Ans.* 4° 6′.

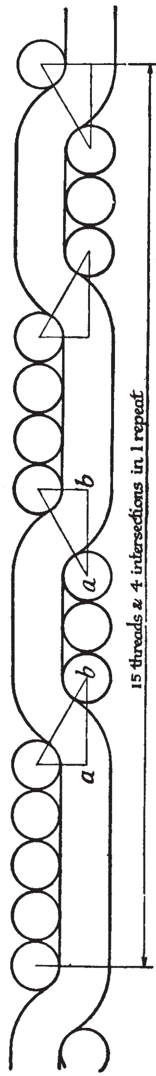


Fig. 17

9. A cylindrical boiler of the Lancashire type is 30 ft. long by 9 ft. diameter; it has two flues 3 ft. 5 in. diameter, running through it from end to end. Calculate the approximate number of gallons of water it will hold, if the working level of the water is 2 ft. from the top. *Ans.* 6516 gall.

10. The trough of a dyeing jigger is 4 ft. wide by 2 ft. deep, while its parallel sides are in the form of a symmetrical trapezium measuring 1 ft. 9 in. at the bottom and 3 ft. 6 in. at the top. Calculate the number of gallons of dyeing liquor it will hold when filled to within 6 in. of the top. ($6\frac{1}{4}$ gall. liquor per cubic foot.) *Ans.* 90·235 gall.

11. The hypotenuse and perpendicular of a right-angled triangle are respectively $4\frac{1}{2}$ ft. and $2\frac{1}{2}$ ft. Find the sine of the angle and the angle itself.

Ans. ·5555. $33^\circ\ 45'$.

12. If in a circle of 9 ft. diameter, a chord equals 7·484 ft., and a radius is drawn to one end of the chord, find the cosine of angle subtended by the above radius and another radius parallel to the chord. *Ans.* $33^\circ\ 45'$.

13. If in a triangle the value of the cosine of an angle is ·8316, what is the actual length of the base of such a triangle when the hypotenuse is $4\frac{1}{2}$ ft.?

Ans. 3·7422 ft.

14. Seeing that the sum of the angles in a triangle is 2 right angles, deduce a formula for expressing the area of a

regular octagon in terms of its side. Use your result to check question 7, Part I, Chap. VIII.

Ans. $4{\cdot}8284b^2$, where b = side.

Chapter VIII, pp. 74–90

1. Jute yarn counts are expressed in pounds per spyndle of 14,400 yards, while certain continental yarn counts are in hundreds of metres per kilogram. Take 1 m. = 39·37 in. and 1 kg. = $2\frac{1}{5}$ lb., and find an expression to convert continental counts to jute counts. *Ans.* $J = \frac{290}{c}$.

2. French cotton counts are reckoned by the number of metres per half kilogram. If a metre is 39·37 in., and a kilogram 2·2046 lb., find the yards per pound in No. 1 French cotton yarn.

Say 1000 m. = 1094 yd. $\frac{1}{2}$ kg. = 1·1023 lb.

Ans. 992·2 yd., say 992.

3. Compare French cotton counts with English cotton counts.

Ans. 992 yards per lb. = No. 1 French,
992 ,, = No. 1·18 English;
∴ English count = 1·18 French count,
or French count $= \frac{\text{English count}}{1{\cdot}18}$.

4. What is the French cotton count for 40^s English?

Ans. $33{\cdot}87^s$ French counts.

5. Calculate the resultant count—no allowance for contraction—when 20^s cotton and 30^s cotton are twisted together. *Ans.* 12^s.

6. If one thread of 20^s cotton were twisted with one thread of 30^s worsted, what would be the resultant cotton count? (Worsted hank = 560 yd., cotton hank = 840 yd.)

Ans. 10^s cotton.

7. A 3-ply linen yarn weighs 1400 gr. per lea of 300 yards. Two of the threads are each 20^s, what is the third? No allowance for contraction; linen counts = number of leas of 300 yards per pound. *Ans.* 10^s.

USEFUL DATA

YARN COUNTS

Cotton: number of hanks of 840 yd. each in 1 lb.
Spun silk: number of hanks of 840 yd. each in 1 lb.
Worsted: number of hanks of 560 yd. each in 1 lb.
Linen: number of leas of 300 yd. each in 1 lb.
Raw silk: number of yards in 1 oz.
Dewsbury woollen: number of yards in 1 oz.
Yorkshire skein woollen: number of yards in 1 dram.
Galashiels woollen: number of cuts of 300 yd. each in 24 oz.
Hawick woollen: number of cuts of 300 yd. each in 26 oz.
Jute: the weight in pounds of 14,400 yd. (1 spyndle).

AVERAGE SPECIFIC GRAVITIES OF COMMON SUBSTANCES

Water ...	... 1·00	Gun-metal ...	... 8·74
Wrought iron	... 7·71	Copper (sheet)	... 8·82
Cast iron ...	... 7·21	Babbit metal	... 7·32
Steel	... 7·87	Zinc (sheet)...	... 7·21
Aluminium ...	... 2·58	Cotton seed oil	... 0·925
Lead (sheet)	... 11·43	Linseed oil ...	... 0·935
Brass ...	... 8·11	Petroleum ...	... 0·878
Zinc	... 7·42	Whale oil ...	... 0·925

WEIGHTS OF COMMON SUBSTANCES

	Lb. per cu. ft.	Lb. per cu. in.
Wrought iron	480	·278
Cast iron	449	·260
Lead (sheet)	712	·412
Steel	490	·284
Aluminium	164	·095
Brass	505	·292
Tin	462	·267
Gun-metal	544	·315
Copper (sheet)	549	·318
Babbit metal	456	·264
Zinc (sheet)	449	·260

DECIMAL EQUIVALENTS OF INCHES UP TO 1 FOOT

Correct to 4 places of Decimals.

Ins.	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
0	—	·0104	·0208	·0313	·0417	·0521	·0625	·0729
1	·0833	·0938	·1042	·1146	·1250	·1354	·1458	·1563
2	·1667	·1771	·1875	·1979	·2083	·2188	·2292	·2396
3	·2500	·2604	·2708	·2813	·2717	·3021	·3125	·3229
4	·3333	·3438	·3542	·3646	·3750	·3854	·3958	·4063
5	·4167	·4271	·4375	·4479	·4583	·4688	·4792	·4896
6	·5000	·5104	·5208	·5313	·5417	·5521	·5625	·5729
7	·5833	·5938	·6042	·6146	·6250	·6354	·6458	·6563
8	·6667	·6771	·6875	·6979	·7083	·7188	·7292	·7396
9	·7500	·7604	·7708	·7813	·7917	·8021	·8125	·8229
10	·8333	·8438	·8542	·8646	·8750	·8854	·8958	·9063
11	·9167	·9271	·9375	·9479	·9583	·9688	·9792	·9896
12	1·0000	—	—	—	—	—	—	—

WHEEL GEARS

Circular pitch $= \dfrac{3{\cdot}1416}{\text{diametral pitch}}$.

Diametral pitch $= \dfrac{3{\cdot}1416}{\text{circular pitch}}$.

Pitch diameter $= \dfrac{\text{No. of teeth}}{\text{diametral pitch}}$.

Pitch diameter $= \dfrac{\text{No. of teeth} \times \text{circular pitch}}{3{\cdot}1416}$.

Outside diameter of wheel $= \dfrac{\text{No. of teeth} + 2}{\text{diametral pitch}}$.

Outside diameter of wheel $= \dfrac{(\text{No. of teeth} + 2) \times \text{circular pitch}}{3{\cdot}1416}$.

Number of teeth = pitch diam. $\times$ diametral pitch.

Number of teeth $= \dfrac{\text{pitch diam.} \times 3{\cdot}1416}{\text{circular pitch}}$.

Distance between centres of two wheels in gear $= \dfrac{\text{sum of No. of teeth in both wheels}}{2 \times \text{diametral pitch}}$.

Distance between centres of two wheels in gear $= \dfrac{\text{sum of pitch diam. of both wheels}}{2}$.

COMMON FUNCTIONS OF π

π =	3·1416.	$\frac{\pi}{7}$ =	·4488.	π^2 =	9·8696.	$\sqrt{\pi}$ =	1·7725.
$\frac{\pi}{2}$ =	1·5708.	$\frac{\pi}{16}$ =	·1963.	π^3 =	31·0063.	$\sqrt[3]{\pi}$ =	1·4646.
$\frac{\pi}{3}$ =	1·0472.	$\frac{\pi}{24}$ =	·1309.	$\frac{1}{\pi}$ =	·3183.	$\frac{1}{\sqrt{\pi}}$ =	·5642.
$\frac{\pi}{4}$ =	·7854.	$\frac{\pi}{32}$ =	·0982.	$\frac{1}{\pi^2}$ =	·1013.	$\frac{1}{\sqrt[3]{\pi}}$ =	·6828.
$\frac{\pi}{6}$ =	·5236.	$\frac{\pi}{180}$ =	·0175.	$\frac{1}{\pi^3}$ =	·0323.	$\log \pi$ =	·4971.

WEIGHTS AND VOLUMES OF WATER

1 gall. =	277·274 cu. in. =	·16 cu. ft. =	10 lb.
·003607 ,, =	1·000 ,, =	·00058 ,, =	·03607 ,,
6·23 ,, =	1728 ,, =	1·00 ,, =	62·35 ,,
·1 ,, =	27·73 ,, =	·01605 ,, =	1·00 ,,

STANDARD WEIGHTS AND MEASURES

LENGTH

ml.	fur.	po.	yd.	ft.	in.
1 =	8 =	320 =	1760 =	5280 =	63360
	1 =	40 =	220 =	660 =	7920
		1 =	5½ =	16·5 =	198
			1 =	3 =	36
				1 =	12

SURFACE

sq. ml.	ac.	ro.	sq. po.	sq. yd.	sq. ft.	sq. in.
1 =	640 =	2560 =	102400 =	3097600 =	27878400 =	4014489600
	1 =	4 =	160 =	4840 =	43560 =	6272640
		1 =	40 =	1210 =	10890 =	1568160
			1 =	30¼ =	272¼ =	39204
				1 =	9 =	1296
					1 =	144

VOLUME

cu. yd.	cu. ft.	cu. in.
1 =	27 =	46656
	1 =	1728

CAPACITY

qr.	bus.	pk.	gall.	qt.	pt.	gills	approx. cu. in.
1 =	8 =	32 =	64 =	256 =	512 =	2048 =	17758
	1 =	4 =	8 =	32 =	64 =	256 =	2220
		1 =	2 =	8 =	16 =	64 =	555
			1 =	4 =	8 =	32 =	277
				1 =	2 =	8 =	69
					1 =	4 =	35
						1 =	8·7

WEIGHT

ton	cwt.	qr.	st.	lb.	oz.	dr.	gr.
1 =	20 =	80 =	160 =	2240 =	35840 =	573440 =	15680000
	1 =	4 =	8 =	112 =	1792 =	28672 =	784000
		1 =	2 =	28 =	448 =	7168 =	196000
			1 =	14 =	224 =	3584 =	98000
				1 =	16 =	256 =	7000
					1 =	16 =	437½
						1 =	27·34

SUMMARY OF MENSURATION RULES

(The page numbers refer to Part I of *Textile Mathematics*)

Page		
19.	Square of side s.	Area $= s^2$ square units.
19.	Rectangle: length l. breadth b.	,, $= lb$,,
28.	Triangle: base b. altitude a.	,, $= \frac{ab}{2}$,,
30.	Triangle: sides a, b, c. semi-perimeter s.	,, $= \sqrt{s(s-a)(s-b)(s-c)}$ square units.
79.	(equilateral) side a.	,, $= \frac{a^2\sqrt{3}}{4}$ square units.
33.	Parallelogram: base b. perpendicular height h.	,, $= bh$ square units.
33.	Rhombus: perpendicular height h.	,, $= bh$,,
35.	Trapezium: side x parallel to side y. d perpendicular distance between x and y.	,, $= d\left(\frac{x+y}{2}\right)$ square units.
36.	Quadrilateral: diagonal d. offsets to opposite vertices a and b.	,, $= \frac{d}{2}(a+b)$ square units.

Page		
39.	Regular polygon: n sides each b units long, and distance from centre of side to centre of polygon a.	Area $= \frac{nab}{2}$ square units.
46.	Circle: radius r.	,, $= \pi r^2$ square units.
47.	Circle: diameter d.	,, $= \frac{\pi}{4}d^2$,,
57.	Arc of circle: diameter d, subtending angle of D degrees at centre.	Length $= \cdot 00872 d$ D units.
59.	Chord: $2c$.	$\frac{\text{Length}}{2} = c = \sqrt{f^2 - a^2}$.
62.	Sector: diameter of circle d. angle of D degrees subtended at centre.	Area $= \cdot 00218$D d^2 square units.
71. / 75.	Rectangular solids: length l. breadth b. height h.	Volume $= lbh$ cubic units. Surface area $= 2(lh + lb + bh)$ square units.
75.	Cube: side s.	Volume $= s^3$ cubic units. Surface area $= 6s^2$ square units.

Summary of Mensuration Rules (*Continued*)

Page	Solid	Rules
78.	Regular prisms: height h.	Volume = (area of base $\times h$) cubic units.
77.	perimeter p.	Surface area = (area of 2 ends $+ p \times h$) sq. units.
80.	Cylinder: diameter d.	Volume $= \frac{\pi}{4} d^2 l$ cubic units.
80.	radius r.	,, $= \pi r^2 l$,,
80.	length l.	Surface area $= \pi d(l + r)$ square units.
86.	Pyramid: height h.	Volume $= \frac{\text{area of base}}{3} h$ cubic units.
	perimeter of base, p.	
86.	slant height s.	Surface area $= \frac{ps}{2}$ square units.
90.	Cone: height h.	Volume $= \cdot 2618 D^2 h$ cubic units.
90.	perimeter of base p.	Curved surface area $= \pi r s$ square units.
91.		,, ,, $= \pi r \sqrt{h^2 + r^2}$ square units.
91.	slant height s.	Whole surface area $= \pi r(r + \sqrt{h^2 + r^2})$ sq. units.
94.	Frustum of pyramid: area of larger end A. area of smaller end a.	Volume $= \frac{t}{3}(A + \sqrt{Aa} + a)$ cubic units.
95.	perimeter of larger end, P. perimeter of smaller end p. slant height or thickness T. vertical height or thickness t.	Area of slant surfaces $= \frac{P + p}{2} T$ square units.

Page	Solid	Rules
96.	Frustum of cone: radius of larger end R, radius of smaller end r, slant height or thickness T. vertical height or thickness t.	Volume $= \frac{\pi t}{3}(R^2 + Rr + r^2)$ cubic units. Area of curved surface $= \pi T(R + r)$ square units.
99.	Sphere: radius r.	Volume $= \frac{4}{3}\pi r^3 = \frac{\pi}{6} D^3$ cubic units.
99.	diameter D.	Surface area $= 4\pi r^2 = \pi D^2$ square units.
101.	Segment of sphere: radius r. diameter d.	Volume $= \frac{\pi h^2}{6}(3D - 2h)$ cubic units.
100.	height h.	Area of curved surface $= 2\pi R h$ square units.
102.	Zone of sphere: area of one circular end r_1. area of other circular end r_2. thickness of zone t. radius of complete sphere R. diameter of complete sphere D.	Volume $= \frac{\pi t}{6}\{3(r_1^2 + r_2^2) + t^2\}$ cubic units. Area of curved surface $= \pi D t$ square units.

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	26
							1 31	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTI LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTI LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NATURAL SINES

Degree.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1′	2′	3′	4′	5′
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10

NATURAL SINES

Degree.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1′	2′	3′	4′	5′
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000	0	0	0	0	0
90	1·000														

TABLE IV

NATURAL TANGENTS

Degree.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences.				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30

NATURAL TANGENTS

Degree.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences.				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	$\overline{0}057$	$\overline{0}145$	$\overline{0}233$	$\overline{0}323$	$\overline{0}413$	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	$\overline{0}061$	$\overline{0}237$	$\overline{0}415$	$\overline{0}595$	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	214	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646	62	124	186	248	310
78	4·7046	7453	7867	8288	8716	9152	9594	$\overline{0}045$	$\overline{0}504$	$\overline{0}970$	73	146	219	292	365
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140	87	175	262	350	437
80	5·6713	7297	7894	8502	9124	9758	$\overline{0}405$	1066	$\overline{1}742$	$\overline{2}432$					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	$\overline{0}264$					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	$\overline{0}285$	*Mean* differences no longer sufficiently accurate.				
83	8·1443	2636	3863	5126	6427	7769	9152	$\overline{0}579$	$\overline{2}052$	$\overline{3}572$					
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					
90	∞														

TABLE V

LOGARITHMIC SINES

Degree.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1′	2′	3′	4′	5′
0	$-\infty$	7·2419	5429	7190	8439	9408	$\overline{0}200$	$\overline{0}870$	$\overline{1}450$	$\overline{1}961$					
1	8·2419	2832	3210	3558	3880	4179	4459	4723	4971	5206					
2	8·5428	5640	5842	6035	6220	6397	6567	6731	6889	7041					
3	8·7188	7330	7468	7602	7731	7857	7979	8098	8213	8326	21	41	62	82	103
4	8·8436	8543	8647	8749	8849	8946	9042	9135	9226	9315	16	32	48	64	80
5	8·9403	9489	9573	9655	9736	9816	9894	9970	$\overline{0}046$	$\overline{0}120$	13	26	39	52	65
6	9·0192	0264	0334	0403	0472	0539	0605	0670	0734	0797	11	22	33	44	55
7	9·0859	0920	0981	1040	1099	1157	1214	1271	1326	1381	10	19	29	38	48
8	9·1436	1489	1542	1594	1646	1697	1747	1797	1847	1895	8	17	25	34	42
9	9·1943	1991	2038	2085	2131	2176	2221	2266	2310	2353	8	15	23	30	38
10	9·2397	2439	2482	2524	2565	2606	2647	2687	2727	2767	7	14	20	27	34
11	9·2806	2845	2883	2921	2959	2997	3034	3070	3107	3143	6	12	19	25	31
12	9·3179	3214	3250	3284	3319	3353	3387	3421	3455	3488	6	11	17	23	28
13	9·3521	3554	3586	3618	3650	3682	3713	3745	3775	3806	5	11	16	21	26
14	9·3837	3867	3897	3927	3957	3986	4015	4044	4073	4102	5	10	15	20	24
15	9·4130	4158	4186	4214	4242	4269	4296	4323	4350	4377	5	9	14	18	23
16	9·4403	4430	4456	4482	4508	4533	4559	4584	4609	4634	4	9	13	17	21
17	9·4659	4684	4709	4733	4757	4781	4805	4829	4853	4876	4	8	12	16	20
18	9·4900	4923	4946	4969	4992	5015	5037	5060	5082	5104	4	8	11	15	19
19	9·5126	5148	5170	5192	5213	5235	5256	5278	5299	5320	4	7	11	14	18
20	9·5341	5361	5382	5402	5423	5443	5463	5484	5504	5523	3	7	10	14	17
21	9·5543	5563	5583	5602	5621	5641	5660	5679	5698	5717	3	6	10	13	16
22	9·5736	5754	5773	5792	5810	5828	5847	5865	5883	5901	3	6	9	12	15
23	9·5919	5937	5954	5972	5990	6007	6024	6042	6059	6076	3	6	9	12	15
24	9·6093	6110	6127	6144	6161	6177	6194	6210	6227	6243	3	6	8	11	14
25	9·6259	6276	6292	6308	6324	6340	6356	6371	6387	6403	3	5	8	11	13
26	9·6418	6434	6449	6465	6480	6495	6510	6526	6541	6556	3	5	8	10	13
27	9·6570	6585	6600	6615	6629	6644	6659	6673	6687	6702	2	5	7	10	12
28	9·6716	6730	6744	6759	6773	6787	6801	6814	6828	6842	2	5	7	9	12
29	9·6856	6869	6883	6896	6910	6923	6937	6950	6963	6977	2	4	7	9	11
30	9·6990	7003	7016	7029	7042	7055	7068	7080	7093	7106	2	4	6	9	11
31	9·7118	7131	7144	7156	7168	7181	7193	7205	7218	7230	2	4	6	8	10
32	9·7242	7254	7266	7278	7290	7302	7314	7326	7338	7349	2	4	6	8	10
33	9·7361	7373	7384	7396	7407	7419	7430	7442	7453	7464	2	4	6	8	10
34	9·7476	7487	7498	7509	7520	7531	7542	7553	7564	7575	2	4	6	7	9
35	9·7586	7597	7607	7618	7629	7640	7650	7661	7671	7682	2	4	5	7	9
36	9·7692	7703	7713	7723	7734	7744	7754	7764	7774	7785	2	3	5	7	9
37	9·7795	7805	7815	7825	7835	7844	7854	7864	7874	7884	2	3	5	7	8
38	9·7893	7903	7913	7922	7932	7941	7951	7960	7970	7979	2	3	5	6	8
39	9·7989	7998	8007	8017	8026	8035	8044	8053	8063	8072	2	3	5	6	8
40	9·8081	8090	8099	8108	8117	8125	8134	8143	8152	8161	1	3	4	6	7
41	9·8169	8178	8187	8195	8204	8213	8221	8230	8238	8247	1	3	4	6	7
42	9·8255	8264	8272	8280	8289	8297	8305	8313	8322	8330	1	3	4	6	7
43	9·8338	8346	8354	8362	8370	8378	8386	8394	8402	8410	1	3	4	5	7
44	9·8418	8426	8433	8441	8449	8457	8464	8472	8480	8487	1	3	4	5	6
45	9·8495	8502	8510	8517	8525	8532	8540	8547	8555	8562	1	2	4	5	6

Logarithmic sine = true logarithm of sine + 10.
Thus L. sin 1° = log. (sin 1°) + 10
= $\overline{2}\cdot2419 + 10$
= 8·2419.

LOGARITHMIC SINES

Degree.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1′	2′	3′	4′	5′
45	9·8495	8502	8510	8517	8525	8532	8540	8547	8555	8562	1	2	4	5	6
46	9·8569	8577	8584	8591	8598	8606	8613	8620	8627	8634	1	2	4	5	6
47	9·8641	8648	8655	8662	8669	8676	8683	8690	8697	8704	1	2	3	5	6
48	9·8711	8718	8724	8731	8738	8745	8751	8758	8765	8771	1	2	3	4	6
49	9·8778	8784	8791	8797	8804	8810	8817	8823	8830	8836	1	2	3	4	5
50	9·8843	8849	8855	8862	8868	8874	8880	8887	8893	8899	1	2	3	4	5
51	9·8905	8911	8917	8923	8929	8935	8941	8947	8953	8959	1	2	3	4	5
52	9·8965	8971	8977	8983	8989	8995	9000	9006	9012	9018	1	2	3	4	5
53	9·9023	9029	9035	9041	9046	9052	9057	9063	9069	9074	1	2	3	4	5
54	9·9080	9085	9091	9096	9101	9107	9112	9118	9123	9128	1	2	3	4	5
55	9·9134	9139	9144	9149	9155	9160	9165	9170	9175	9181	1	2	3	3	4
56	9·9186	9191	9196	9201	9206	9211	9216	9221	9226	9231	1	2	3	3	4
57	9·9236	9241	9246	9251	9255	9260	9265	9270	9275	9279	1	2	2	3	4
58	9·9284	9289	9294	9298	9303	9308	9312	9317	9322	9326	1	2	2	3	4
59	9·9331	9335	9340	9344	9349	9353	9358	9362	9367	9371	1	1	2	3	4
60	9·9375	9380	9384	9388	9393	9397	9401	9406	9410	9414	1	1	2	3	4
61	9·9418	9422	9427	9431	9435	9439	9443	9447	9451	9455	1	1	2	3	3
62	9·9459	9463	9467	9471	9475	9479	9483	9487	9491	9495	1	1	2	3	3
63	9·9499	9503	9507	9510	9514	9518	9522	9525	9529	9533	1	1	2	3	3
64	9·9537	9540	9544	9548	9551	9555	9558	9562	9566	9569	1	1	2	2	3
65	9·9573	9576	9580	9583	9587	9590	9594	9597	9601	9604	1	1	2	2	3
66	9·9607	9611	9614	9617	9621	9624	9627	9631	9634	9637	1	1	2	2	3
67	9·9640	9643	9647	9650	9653	9656	9659	9662	9666	9669	1	1	2	2	3
68	9·9672	9675	9678	9681	9684	9687	9690	9693	9696	9699	0	1	1	2	2
69	9·9702	9704	9707	9710	9713	9716	9719	9722	9724	9727	0	1	1	2	2
70	9·9730	9733	9735	9738	9741	9743	9746	9749	9751	9754	0	1	1	2	2
71	9·9757	9759	9762	9764	9767	9770	9772	9775	9777	9780	0	1	1	2	2
72	9·9782	9785	9787	9789	9792	9794	9797	9799	9801	9804	0	1	1	2	2
73	9·9806	9808	9811	9813	9815	9817	9820	9822	9824	9826	0	1	1	2	2
74	9·9828	9831	9833	9835	9837	9839	9841	9843	9845	9847	0	1	1	1	2
75	9·9849	9851	9853	9855	9857	9859	9861	9863	9865	9867	0	1	1	1	2
76	9·9869	9871	9873	9875	9876	9878	9880	9882	9884	9885	0	1	1	1	2
77	9·9887	9889	9891	9892	9894	9896	9897	9899	9901	9902	0	1	1	1	1
78	9·9904	9906	9907	9909	9910	9912	9913	9915	9916	9918	0	1	1	1	1
79	9·9919	9921	9922	9924	9925	9927	9928	9929	9931	9932	0	0	1	1	1
80	9·9934	9935	9936	9937	9939	9940	9941	9943	9944	9945	0	0	1	1	1
81	9·9946	9947	9949	9950	9951	9952	9953	9954	9955	9956	0	0	1	1	1
82	9·9958	9959	9960	9961	9962	9963	9964	9965	9966	9967	0	0	1	1	1
83	9·9968	9968	9969	9970	9971	9972	9973	9974	9975	9975	0	0	0	1	1
84	9·9976	9977	9978	9978	9979	9980	9981	9981	9982	9983	0	0	0	0	1
85	9·9983	9984	9985	9985	9986	9987	9987	9988	9988	9989	0	0	0	0	0
86	9·9989	9990	9990	9991	9991	9992	9992	9993	9993	9994	0	0	0	0	0
87	9·9994	9994	9995	9995	9996	9996	9996	9996	9997	9997	0	0	0	0	0
88	9·9997	9998	9998	9998	9998	9999	9999	9999	9999	9999	0	0	0	0	0
89	9·9999	9999	$\overline{0}000$	$\overline{0}000$	$\overline{0}000$	$\overline{0}000$	$\overline{0}000$	$\overline{0}000$	$\overline{0}000$	$\overline{0}000$	0	0	0	0	0
90	10·0000														

LOGARITHMIC TANGENTS

Degree.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences.				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
0	−∞	7·2419	5429	7190	8439	9409	$\bar{0}$200	$\bar{0}$870	$\bar{1}$450	$\bar{1}$962					
1	8·2419	2833	3211	3559	3881	4181	4461	4725	4973	5208					
2	8·5431	5643	5845	6038	6223	6401	6571	6736	6894	7046	29	58	87	116	145
3	8·7194	7337	7475	7609	7739	7865	7988	8107	8223	8336	21	41	62	83	103
4	8·8446	8554	8659	8762	8862	8960	9056	9150	9241	9331	16	32	48	64	81
5	8·9420	9506	9591	9674	9756	9836	9915	9992	$\bar{0}$068	$\bar{0}$143	13	26	40	53	66
6	9·0216	0289	0360	0430	0499	0567	0633	0699	0764	0828	11	22	34	45	56
7	9·0891	0954	1015	1076	1135	1194	1252	1310	1367	1423	10	20	29	39	49
8	9·1478	1533	1587	1640	1693	1745	1797	1848	1898	1948	9	17	26	35	43
9	9·1997	2046	2094	2142	2189	2236	2282	2328	2374	2419	8	16	23	31	39
10	9·2463	2507	2551	2594	2637	2680	2722	2764	2805	2846	7	14	21	28	35
11	9·2887	2927	2967	3006	3046	3085	3123	3162	3200	3237	6	13	19	26	32
12	9·3275	3312	3349	3385	3422	3458	3493	3529	3564	3599	6	12	18	24	30
13	9·3634	3668	3702	3736	3770	3804	3837	3870	3903	3935	6	11	17	22	28
14	9·3968	4000	4032	4064	4095	4127	4158	4189	4220	4250	5	10	16	21	26
15	9·4281	4311	4341	4371	4400	4430	4459	4488	4517	4546	5	10	15	20	25
16	9·4575	4603	4632	4660	4688	4716	4744	4771	4799	4826	5	9	14	19	23
17	9·4853	4880	4907	4934	4961	4987	5014	5040	5066	5092	4	9	13	18	22
18	9·5118	5143	5169	5195	5220	5245	5270	5295	5320	5345	4	8	13	17	21
19	9·5370	5394	5419	5443	5467	5491	5516	5539	5563	5587	4	8	12	16	20
20	9·5611	5634	5658	5681	5704	5727	5750	5773	5796	5819	4	8	12	15	19
21	9·5842	5864	5887	5909	5932	5954	5976	5998	6020	6042	4	7	11	15	19
22	9·6064	6086	6108	6129	6151	6172	6194	6215	6236	6257	4	7	11	14	18
23	9·6279	6300	6321	6341	6362	6383	6404	6424	6445	6465	3	7	10	14	17
24	9·6486	6506	6527	6547	6567	6587	6607	6627	6647	6667	3	7	10	13	17
25	9·6687	6706	6726	6746	6765	6785	6804	6824	6843	6863	3	7	10	13	16
26	9·6882	6901	6920	6939	6958	6977	6996	7015	7034	7053	3	6	9	13	16
27	9·7072	7090	7109	7128	7146	7165	7183	7202	7220	7238	3	6	9	12	15
28	9·7257	7275	7293	7311	7330	7348	7366	7384	7402	7420	3	6	9	12	15
29	9·7438	7455	7473	7491	7509	7526	7544	7562	7579	7597	3	6	9	12	15
30	9·7614	7632	7649	7667	7684	7701	7719	7736	7753	7771	3	6	9	12	14
31	9·7788	7805	7822	7839	7856	7873	7890	7907	7924	7941	3	6	9	11	14
32	9·7958	7975	7992	8008	8025	8042	8059	8075	8092	8109	3	6	8	11	14
33	9·8125	8142	8158	8175	8191	8208	8224	8241	8257	8274	3	5	8	11	14
34	9·8290	8306	8323	8339	8355	8371	8388	8404	8420	8436	3	5	8	11	14
35	9·8452	8468	8484	8501	8517	8533	8549	8565	8581	8597	3	5	8	11	13
36	9·8613	8629	8644	8660	8676	8692	8708	8724	8740	8755	3	5	8	11	13
37	9·8771	8787	8803	8818	8834	8850	8865	8881	8897	8912	3	5	8	10	13
38	9·8928	8944	8959	8975	8990	9006	9022	9037	9053	9068	3	5	8	10	13
39	9·9084	9099	9115	9130	9146	9161	9176	9192	9207	9223	3	5	8	10	13
40	9·9238	9254	9269	9284	9300	9315	9330	9346	9361	9376	3	5	8	10	13
41	9·9392	9407	9422	9438	9453	9468	9483	9499	9514	9529	3	5	8	10	13
42	9·9544	9560	9575	9590	9605	9621	9636	9651	9666	9681	3	5	8	10	13
43	9·9697	9712	9727	9742	9757	9773	9788	9803	9818	9833	3	5	8	10	13
44	9·9848	9864	9879	9894	9909	9924	9939	9955	9970	9985	3	5	8	10	13
45	10·0000	0015	0030	0045	0061	0076	0091	0106	0121	0136	3	5	8	10	13

Logarithmic tangent = true logarithm of tangent + 10.
Thus L. tan 1° = log, (tan 1°) + 10
= $\bar{2}$·2419 + 10
= 8·2419.

TABLE VI

LOGARITHMIC TANGENTS

Degree.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences.				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
45	10·0000	0015	0030	0045	0061	0076	0091	0106	0121	0136	3	5	8	10	13
46	10·0152	0167	0182	0197	0212	0228	0243	0258	0273	0288	3	5	8	10	13
47	10·0303	0319	0334	0349	0364	0379	0395	0410	0425	0440	3	5	8	10	13
48	10·0456	0471	0486	0501	0517	0532	0547	0562	0578	0593	3	5	8	10	13
49	10·0608	0624	0639	0654	0670	0685	0700	0716	0731	0746	3	5	8	10	13
50	10·0762	0777	0793	0808	0824	0839	0854	0870	0885	0901	3	5	8	10	13
51	10·0916	0932	0947	0963	0978	0994	1010	1025	1041	1056	3	5	8	10	13
52	10·1072	1088	1103	1119	1135	1150	1166	1182	1197	1213	3	5	8	10	13
53	10·1229	1245	1260	1276	1292	1308	1324	1340	1356	1371	3	5	8	11	13
54	10·1387	1403	1419	1435	1451	1467	1483	1499	1516	1532	3	5	8	11	13
55	10·1548	1564	1580	1596	1612	1629	1645	1661	1677	1694	3	5	8	11	14
56	10·1710	1726	1743	1759	1776	1792	1809	1825	1842	1858	3	5	8	11	14
57	10·1875	1891	1908	1925	1941	1958	1975	1992	2008	2025	3	6	8	11	14
58	10·2042	2059	2076	2093	2110	2127	2144	2161	2178	2195	3	6	9	11	14
59	10·2212	2229	2247	2264	2281	2299	2316	2333	2351	2368	3	6	9	12	14
60	10·2386	2403	2421	2438	2456	2474	2491	2509	2527	2545	3	6	9	12	15
61	10·2562	2580	2598	2616	2634	2652	2670	2689	2707	2725	3	6	9	12	15
62	10·2743	2762	2780	2798	2817	2835	2854	2872	2891	2910	3	6	9	12	15
63	10·2928	2947	2966	2985	3004	3023	3042	3061	3080	3099	3	6	9	13	16
64	10·3118	3137	3157	3176	3196	3215	3235	3254	3274	3294	3	6	10	13	16
65	10·3313	3333	3353	3373	3393	3413	3433	3453	3473	3494	3	7	10	13	17
66	10·3514	3535	3555	3576	3596	3617	3638	3659	3679	3700	3	7	10	14	17
67	10·3721	3743	3764	3785	3806	3828	3849	3871	3892	3914	4	7	11	14	18
68	10·3936	3958	3980	4002	4024	4046	4068	4091	4113	4136	4	7	11	15	19
69	10·4158	4181	4204	4227	4250	4273	4296	4319	4342	4366	4	8	12	15	19
70	10·4389	4413	4437	4461	4484	4509	4533	4557	4581	4606	4	8	12	16	20
71	10·4630	4655	4680	4705	4730	4755	4780	4805	4831	4857	4	8	13	17	21
72	10·4882	4908	4934	4960	4986	5013	5039	5066	5093	5120	4	9	13	18	22
73	10·5147	5174	5201	5229	5256	5284	5312	5340	5368	5397	5	9	14	19	23
74	10·5425	5454	5483	5512	5541	5570	5600	5629	5659	5689	5	10	15	20	25
75	10·5719	5750	5780	5811	5842	5873	5905	5936	5968	6000	5	10	16	21	26
76	10·6032	6065	6097	6130	6163	6196	6230	6264	6298	6332	6	11	17	22	28
77	10·6366	6401	6436	6471	6507	6542	6578	6615	6651	6688	6	12	18	24	30
78	10·6725	6763	6800	6838	6877	6915	6954	6994	7033	7073	6	13	19	26	32
79	10·7113	7154	7195	7236	7278	7320	7363	7406	7449	7493	7	14	21	28	35
80	10·7537	7581	7626	7672	7718	7764	7811	7858	7906	7954	8	16	23	31	39
81	10·8003	8052	8102	8152	8203	8255	8307	8360	8413	8467	9	17	26	35	43
82	10·8522	8577	8633	8690	8748	8806	8865	8924	8985	9046	10	20	29	39	49
83	10·9109	9172	9236	9301	9367	9433	9501	9570	9640	9711	11	22	34	45	56
84	10·9784	9857	9932	$\bar{0}$008	$\bar{0}$085	$\bar{0}$164	$\bar{0}$244	$\bar{0}$326	$\bar{0}$409	$\bar{0}$494	13	26	40	53	66
85	11·0580	0669	0759	0850	0944	1040	1138	1238	1341	1446	16	32	48	64	81
86	11·1554	1664	1777	1893	2012	2135	2261	2391	2525	2663	20	41	62	83	103
87	11·2806	2954	3106	3264	3429	3599	3777	3962	4155	4357	29	58	87	116	144
88	11·4569	4792	5027	5275	5539	5819	6119	6441	6789	7167					
89	11·7581	8038	8550	9130	9800	$\bar{0}$591	$\bar{1}$561	$\bar{2}$810	$\bar{4}$571	$\bar{7}$581					
90	+∞														

PRINTED AND BOUND IN GREAT BRITAIN
BY BLACKIE & SON, LTD., GLASGOW.